GATE Previous Years' Questions

1. A quantity of water is completely converted into steam by boiling. **[GATE 1991]**
(a) This is a second order phase transition.
(b) At the boiling point, the chemical potential in the vapour phase is less than that in the liquid phase.
(c) The boiling temperature would increase if the boiling were done under increased pressure.
(d) The entropy of the steam is greater than that of the water.

2. The ratio $\gamma = C_P / C_V$ of H_2 gas is 7/5 at room temperature, and 5/3 at low temperatures (around 50 K). The reason for this change is **[GATE 1992]**
(a) Dissociation of H_2 molecules (b) Suppression of rotational degrees of freedom
(c) Formation fo random clusters of molecules. (d) Liquefaction of the gas.

3. If $\overline{v}$, v_p and v_{rms} denote the average, most probable, and root mean square values respectively of the molecular speeds of a gas at room temperature being Maxwellian velocity distribution, then **[GATE 1994]**
(a) $v_{rms} < \overline{v} < v_p$ (b) $v_{rms} < v_p < \overline{v}$ (c) $\overline{v} < v_{rms} < v_p$ (d) $v_p < \overline{v} < v_{rms}$

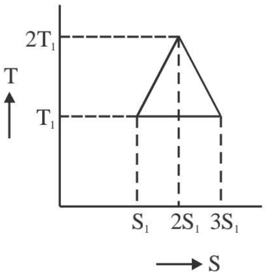
4. An ideal gas in a cylider is compressed adiabatically to one-third of its initial volume. During this process 20 J work is done on the gas by compressing agent. Which of the following statements is true for this case?
(a) Change in the internal energy in this process is zero. **[GATE 1997]**
(b) The internal energy increases by 20 J
(c) The internal energy decreases by 20 J
(d) Temperature of the gas decreases.

5. Mean total energy of a classical three-dimensional harmonical oscillator in equilibrium with a heat reservoir at temperature T is: **[GATE 1997]**
(a) $k_B T$ (b) $\frac{3}{2}k_B T$ (c) $2\,k_B T$ (d) $3\,k_B T$

6. The root-mean square speed of a particle of mass m in the kinetic theory is given by **[GATE 1997]**
(a) $\sqrt{\frac{k_B T}{m}}$ (b) $\sqrt{\frac{2k_B T}{m}}$ (c) $\sqrt{\frac{3k_B T}{m}}$ (d) $\sqrt{\frac{8k_B T}{m}}$

7. The sublimation curve of solid ammonia is given by $\ln p = 23 - 3750/T$ and the vaporization curve of the liquid ammonia is given by $\ln p = 19.5 - 3050/T$, where p is in mm of Hg and T is in K. The temperature of the triple point of ammonia is: **[GATE 1998]**
(a) 3750 K (b) 3050 K (c) 700 K (d) 200 K

8. A reversible engine cycle is shown in the following T-S diagram. The efficiency of the engine is:



(a) 1/3 (b) 1/2 (c) 1/5 (d) 1/4 **[GATE 1999]**

9. An engine absorbs heat at a temperature of 1000 K and rejects heat at 600 K. If the engine operates at maximum possible efficiency, the amount of work performed by the engine, for 2000 J heat input is:
(a) 1600 J (b) 1200 J (c) 800 J (d) 400 J

10. Boyle's law can be expressed in different form as **[GATE 1999]**
(a) $dV/dP = 1$ (b) $dV/dP = V/P$ (c) $dV/dP = P/V$ (d) $dV/dP = -V/P$

11. The boiling point of a liquid at pressure P_0 is T_0. Its molar latent heat of vaporisation is L and molar volume of the liquid phase is negligible as compared to the vapour phase. The vapour phase obeys perfect gas equation. The boiling point T at pressure P is given by **[GATE 1999]**
(a) $P/P_0 = (L/RT_0)(1 - T_0/T)$ (b) $\ln(P/P_0) = (L/RT_0)(1 - T_0/T)$
(c) $\ln(P/P_0) = (L/RT_0)(1 - T/T_0)$ (d) $P/P_0 = 1 + (L/RT_0)\ln(T_0/T)$

12. An amount of heat Q is transferred from a heat reservoir at temperature T_A to another heat reservoir at temperature T_B. What is the change in the entropy ΔS of the combined system? **[GATE 2000]**
(a) $Q\left(\frac{1}{T_B} - \frac{1}{T_A}\right)$ (b) $Q\left(\frac{1}{T_B} + \frac{1}{T_A}\right)$ (c) $\frac{Q}{\sqrt{T_B T_A}} \ln \frac{T_B}{T_A}$ (d) $Q\left(\frac{1}{T_A} - \frac{1}{T_B}\right)$

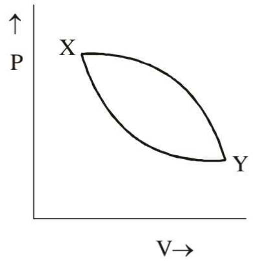
13. Which of the following is an example of a first order phase transition? **[GATE 2000]**
(a) A liquid - gas phase transition at the critical point.
(b) A paramagnet - ferromagnet phase transition
(c) A normal metal - superconductor phase transition.
(d) A liquid - gas phase transition away from the critical point.

14. The mean free path of the particles of a gas at temperature T_0 and pressure p_0 has a value λ_0. If the pressure is increased to 1.5 p_0 and the temperature is reduced to 0.75 T_0, the mean free path **[GATE 2002]**
(a) remains unchanged. (b) is reduced to half
(c) is doubled (d) is equal to 1.125 λ_0

15. A second order phase transition is one in which **[GATE 2003]**
(a) the plot of entropy as a function of temperature shows a discontinuity.
(b) the plot of specific heat as a function of temperature shows a discontinuity
(c) the plot of volume as a function of pressure shows a discontinuity.
(d) the plot of compressibility as a function of temperature is continuous.

16. A sample of ideal gas with initial pressure P and volume V is taken through an isothermal expansion proceed during which the change in entropy is found to be ΔS. The universal gas constant is R. Then the work done by the gas is given by **[GATE 2003]**
(a) $(PV\Delta S)/(nR)$ (b) $nR\Delta S$ (c) PV (d) $(P\Delta S)/(nRV)$

17. A piston containing an ideal gas is originally in the state X (see figure). The gas is taken through a thermal cycle X→Y→X as shown:



The work done by the gas is positive if the direction of the thermal cycle is: **[GATE 2003]**
(a) clockwise (b) counter - clockwise
(c) neither clockwise nor counter-clockwise
(d) clockwise from X→Y and counter-clockwise from Y→X.

18. Which one of the following is a first order phase transition? **[GATE 2004]**
(a) vaporization of a liquid at its boiling point (b) ferromagnetic to paramagnetic
(c) normal liquid He to superfluid He (d) superconducting to normal state

19. In the region of co-existence of a liquid and vapor phases of a material **[GATE 2004]**
(a) C_P and C_V are both infinite
(b) C_V and $\beta = \left[\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P\right]$ are both finite
(c) C_V and $K = \left[-\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T\right]$ are both finite
(d) C_P, β and K are all infnite

20. For any process, the second law of thermodynamics requires that the change of entropy of the universe be **[GATE 2004]**
(a) positive only (b) positive or zero (c) zero only (d) negative or zero

21. If the equation of state for a gas with internal energy U is $pV = \frac{1}{3}U$, then the equation for an adiabatic process is: **[GATE 2005]**
(a) $pV^{1/3} = \text{constant}$ (b) $pV^{2/3} = \text{constant}$ (c) $pV^{4/3} = \text{constant}$ (d) $pV^{3/5} = \text{constant}$

22. The internal energy of n moles of a gas is given $E = \frac{3}{2}nRT - \frac{a}{V}$, where V is the volume of the gas at temperature T and *a* is a positive constant. One mole of the gas is state (T_1, V_1) is allowed to expand adiabatically into vacuum to a final state (T_2, V_2). The temperature T_2 is: **[GATE 2005]**
(a) $T_1 + \frac{a}{R}\left(\frac{1}{V_2} + \frac{1}{V_1}\right)$ (b) $T_1 - \frac{2}{3}\frac{a}{R}\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$ (c) $T_1 + \frac{2}{3}\frac{a}{R}\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$ (d) $T_1 - \frac{1}{3}\frac{a}{R}\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$

23. The mean internal of a one-dimensional classical harmonic oscillator in equilibrium with a heat bath of temperature T is: **[GATE 2006]**
(a) $\frac{1}{2}k_BT$ (b) k_BT (c) $\frac{3}{2}k_BT$ (d) $3k_BT$

24. A heat pump working on the Carnot cycle maintains the inside temperature of a house at 22℃ by suppling 450 kJ s^{-1}. If the outside temperature is 0℃, the heat taken, in kJs^{-1}, from the outside air is approximately.
(a) 487 (b) 470 (c) 467 (d) 417 **[GATE 2007]**

25. The vapour pressure p (in mm of Hg) of a solid, at temperature T, is expressed by $\ell n\ p = 23 - 3863/T$ and that of its liquid phase by $\ell n\ p = 19 - 3063/T$. The triple point (in Kelvin) of the material is:
(a) 185 (b) 190 (c) 195 (d) 200 **[GATE 2007]**

26. The pressure versus temperature diagram of a given system at certain low temperature range is found to be parallel to the temperature axis in the liquid-to-solid transition region. The change in the specific volume remains constant in this region. The conclusion one can get from the above is: **[GATE 2008]**
(a) the entropy of solid is zero in this temperature region.
(b) the entropy increases when the system goes from liquid to solid phase in this temperature region.
(c) the entropy decreases when the system transforms from liquid to solid phase in this region of temperature.
(d) the change in entropy is zero in the liquid-to-solid transition region.

27. Consider a system of N atoms of an ideal gas of type A at temperature T and volume V. It is kept in diffusive contact with another system of N atoms of another ideal gas of type B at the same temperature T and volume V. Once the combined system reaches equilibrium, **[GATE 2008]**
(a) the total entropy of the final system is the same as the sum of the entropy of the individual system always.
(b) the entropy of mixing is $2Nk_B \ell n 2$.
(c) the entropy of the final system is less than that of sum of the initial entropies of the two gases.
(d) the entropy of mixing is non-zero when the atoms A and B are of the same type.

28. Identify which one is a first order phase transition? **[GATE 2009]**
(a) A liquid to gas transition at its critical temperature.
(b) A liquid to gas transition close to its triple point.
(c) A paramegnetic to ferromagnetic transition in the absence of a magnetic field.
(d) A metal to superconductor transition in the absence of a magnetic field.

29. A system of N non-interacting classical point particles is constrained to move on the two-dimensional surface of a sphere. The internal energy of the system is **[GATE 2010]**
(a) $\frac{3}{2}Nk_BT$ (b) $\frac{1}{2}Nk_BT$ (c) Nk_BT (d) $\frac{5}{2}Nk_BT$

30. Which among the following sets of Maxwell relations is correct? (U-internal energy, H-enthalpy, A-Helmholtz free energy and G-Gibbs free energy) **[GATE 2010]**
(a) $T=\left(\frac{\partial U}{\partial V}\right)_S$ and $P=\left(\frac{\partial U}{\partial S}\right)_V$ (b) $V=\left(\frac{\partial H}{\partial P}\right)_S$ and $T=\left(\frac{\partial H}{\partial S}\right)_P$
(c) $P=-\left(\frac{\partial G}{\partial V}\right)_T$ and $V=\left(\frac{\partial G}{\partial P}\right)_S$ (d) $P=-\left(\frac{\partial A}{\partial S}\right)_T$ and $S=-\left(\frac{\partial A}{\partial P}\right)_V$

31. In a first order phase transition, at the transition temperature, specific heat of the system
(a) Diverges and its entropy remains the same **[GATE 2011]**
(b) Diverges and its entropy has finite discontinuity.
(c) Remains unchanged and its entropy has finite discontinuity.
(d) Has finite discontinuity and its entropy diverges.

32. If the isothermal compressibility of a solid is $\kappa_T = 10^{-10}\,(Pa)^{-1}$, the pressure required to increase its density by 1% is approximately. **[GATE 2011]**
(a) $10^4\ Pa$ (b) $10^6\ Pa$ (c) $10^8\ Pa$ (d) $10^{10}\ Pa$

33. The isothermal compressibility, κ of an ideal gas at temperature T_0 and volume V_0, is given by
(a) $-\frac{1}{V_0}\frac{\partial V}{\partial P}\Big|_{T_0}$ (b) $\frac{1}{V_0}\frac{\partial V}{\partial P}\Big|_{T_0}$ (c) $-V_0\frac{\partial P}{\partial V}\Big|_{T_0}$ (d) $V_0\frac{\partial P}{\partial P}\Big|_{T_0}$ **[GATE 2012]**

34. A classical gas of molecules each of mass m, is in thermal equilibrium at the absolute temperature, T. The velocity components of the molecules along the Cartesian axes are v_x, v_y and v_z. The mean value of $(v_x + v_y)^2$ is: **[GATE 2012]**
(a) $\frac{k_BT}{m}$ (b) $\frac{3}{2}\frac{k_BT}{m}$ (c) $\frac{1}{2}\frac{k_BT}{m}$ (d) $\frac{2k_BT}{m}$

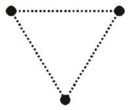
35. At a certain temperature T, the average speed of nitrogen molecules in air is found to be 400 m/s. The most probable and the root mean square speeds of the molecules are, respectively, **[GATE 2012]**
(a) 355 m/s, 434 m/s (b) 820 m/s, 917 m/s
(c) 152 m/s, 301 m/s (d) 422 m/s, 600 m/s

36. If Planck's constant were zero, then the total energy contained in a box filled with radiation of all frequencies at temperature T would be (k is the Boltzmann constant and T is non zero) **[GATE 2013]**

(a) Zero (b) Infinite (c) $\frac{3}{2}kT$ (d) kT

37. N atoms of an ideal gas are enclosed in a container of volume V. The volume of container is changed to 4V, while keeping the total energy constant. The change in the entropy of the gas, in units of $Nk_B \ln 2$, is _______, where k_B is the Boltzmann constant. **[GATE 2016]**

38. Consider a triatomic molecule of the shape shown in the figure below in three dimensions. The heat capacity of this molecule at high temperature (temperature much higher than the vibrational and rotational energy scales of the molecule but lower than its bond dissociation energies) is: **[GATE 2017]**



(a) $\frac{3}{2}k_B$ (b) $3k_B$ (c) $\frac{9}{2}k_B$ (d) $6k_B$

39. A reversible Carnot engine is operated between temperatures T_1 and $T_2\left(T_2 > T_1\right)$ with a photon gas as the working substance. The efficiency of the engine is **[GATE 2017]**

(a) $1-\frac{3T_1}{4T_2}$ (b) $1-\frac{T_1}{T_2}$ (c) $1-\left(\frac{T_1}{T_2}\right)^{3/4}$ (d) $1-\left(\frac{T_1}{T_2}\right)^{4/3}$

ANSWER KEY

1. (c,d)	**2. (b)**	**3. (d)**	**4. (b)**	**5. (d)**	**6. (c)**	**7. (d)**
8. (a)	**9. (c)**	**10. (d)**	**11. (b)**	**12. (a)**	**13. (d)**	**14. (b)**
15. (b)	**16. (a)**	**17. (a)**	**18. (a)**	**19. (d)**	**20. (b)**	**21. (c)**
22. (c)	**23. (b)**	**24. (d)**	**25. (d)**	**26. (d)**	**27. (b)**	**28. (b)**
29. (c)	**30. (b)**	**31. (b)**	**32. (c)**	**33. (a)**	**34. (d)**	**35. (a)**
36. (d)	**37. (2)**	**38. (d)**	**39. (b)**			

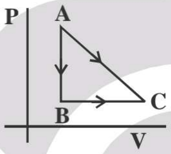
CSIR-UGC-NET Previous Years' Questions

1. Consider the transition of liquid water to steam as water boils at a temperature of 100ºC under a pressure of 1 atmosphere. Which one of the following quantities does not change discontinuously at the transition?
(a) The Gibbs free energy (b) The internal energy **[NET June 2011]**
(c) The entropy (d) The specific volume

2. Consider a Maxwellian distribution of the velocity of the molecules of an ideal gas. Let V_{mp} and V_{rms} denote the most probable velocity and the root mean square velocity, respectively. The magnitude of the ratio V_{mp}/V_{rms} is: **[CSIR Dec. 2011]**
(a) 1 (b) $2/3$ (c) $\sqrt{2/3}$ (d) $3/2$

3. A particle is confined to the region $x \geq 0$ by a potential which increases linearly as $u(x) = u_0 x$. The mean position of the particle at temperature T is: **[CSIR June 2011]**
(a) $\frac{k_B T}{u_0}$ (b) $(k_B T)^2 / u_0$ (c) $\sqrt{\frac{k_B T}{u_0}}$ (d) $u_0 k_B T$

4. A given quantity of gas is taken from the state $A \to C$ reversibly, by two paths, $A \to C$ directly and $A \to B \to C$ as shown in the figure below. **[NET Dec. 2012]**



During the $A \to C$ the work done by the gas is 100 J and the heat absorbed is 150 J. If during the process $A \to B \to C$ the work done by the gas is 30 J, the heat absorbed is:
(a) 20J (b) 80J (c) 220 J (d) 280 J

5. Let ΔW be the work done in a quasistatic reversible thermodynamics process. Which of the following statements about ΔW is correct? **[NET Dec. 2012]**
(a) ΔW is a perfect differential if the process is isothermal
(b) ΔW is a perfect differential if the process is adiabatic
(c) ΔW is always a perfect differential
(d) ΔW cannot be a perfect differential.

6. The pressure *P* of a fluid is related to its number density ρ by the equation of state $P = a\rho + b\rho^2$ where *a* and *b* are constants. If the initial volume of the fluid is V_0, the work done on the system when it is compressed so as to increase the number density from an initial value of ρ_0 to $2\rho_0$ is
(a) $a\rho_0 V_0$ (b) $(a + b\rho_0)\rho_0 V_0$ **[NET Dec. 2014]**
(c) $\left(\frac{3a}{2} + \frac{7\rho_0 b}{3}\right)\rho_0 V_0$ (d) $(a \ln 2 + b\rho_0)\rho_0 V_0$

7. In low density oxygen gas at low temperature, only the translational and rotational modes of the molecules are excited. The specific heat per molecule of the gas is **[CSIR Dec. 2014]**

(a) $\frac{1}{2}k_B$ (b) k_B (c) $\frac{3}{2}k_B$ (d) $\frac{5}{2}k_B$

8. A large number N of Brownian particles in one-dimension start their diffusive motion from the origin at time $t=0$. The diffusion coefficients is D. The number of particles crossing a point at a distance L from the origin, per unit time, depends on L and time t as **[NET June 2015]**

(a) $\frac{N}{\sqrt{4\pi Dt}}e^{-L^2/(4Dt)}$ (b) $\frac{NL}{\sqrt{4\pi Dt}}e^{-4Dt/L^2}$ (c) $\frac{N}{\sqrt{16\pi Dt^3}}e^{-L^2/(4Dt)}$ (d) Ne^{-4Dt/L^2}

9. The specific heat per molecule of a gas of diatomic molecules at high tmeperatures is

(a) $8k_B$ (b) $3.5k_B$ (c) $4.5k_B$ (d) $3k_B$ **[NET June 2016]**

10. When an ideal monatomic gas is expanded adiabatically from an initial volume V_0 to $3V_0$, its temperature changes from T_0 to T. Then the ratio T/T_0 is **[NET June 2016]**

(a) $\frac{1}{3}$ (b) $\left(\frac{1}{3}\right)^{2/3}$ (c) $\left(\frac{1}{3}\right)^{1/3}$ (d) 3

11. The van der Waals equation for one mole of a gas is $\left(p+\frac{a}{V^2}\right)(V-b)=RT$. The corresponding equation of state for n moles of this gas at pressure p, volume V and temperature T, is **[NET June 2018]**

(a) $\left(p+\frac{an^2}{V^2}\right)(V-nb)=nRT$ (b) $\left(p+\frac{a}{V^2}\right)(V-nb)=nRT$

(c) $\left(p+\frac{an^2}{V^2}\right)(V-nb)=RT$ (d) $\left(p+\frac{a}{V^2}\right)(V-nb)=RT$

12. The equation of state of an ideal gas is $pV=RT$. At very low temperatures, the volume expansion coefficient $\frac{1}{V}\frac{\partial V}{\partial T}$ at constant pressure **[NET June 2019]**

(a) diverges as $1/T^2$ (b) diverges as $1/T$ (c) vanishes as T (d) is independent of the temperature

13. An ideal Carnot engine extracts 100 J from a heat source and dumps 40 J to a heat sink at 300 K. The temperature of the heat source is **[NET Dec. 2019]**

(a) 600 K (b) 700 K (c) 750 K (d) 650 K

ANSWER KEY

1. (a) 2. (c) 3. (a) 4. (b) 5. (b) 6. (d) 7. (d)

8. (*) 9. (b,c) 10. (b) 11. (a) 12. (b) 13. (c)

JEST Previous Years' Questions

1. Efficiency of a perfectly reversible (Carrot) heat engine operating between absolute temperature T and zero is equal to **[JEST 2012]**
(a) 0 (b) 0.5 (c) 0.75 (d) 1

2. A monoatomic ideal gas at 17°C is adiabatically compressed to 1/8 of its original volume. The temperature after compression is **[JEST 2012]**
(a) 2.1°C (b) 17°C (c) –200.5°C (d) 887° C

3. A thermally insulated ideal gas of volume V_1 and temperature T expands to another enclosure of volume V_2 through a prous plug. What is the change in the temperature of the gas? **[JEST 2012]**
(a) 0 (b) $T\ln\frac{V_1}{V_2}$ (c) $T\ln\frac{V_2}{V_1}$ (d) $T\ln\left[(V_2-V_1)/V_2\right]$

4. For a diatomic ideal gas near room temperature, what fraction of the heat supplied is available for external work if the gas is expanded at constant pressure? **[JEST 2013]**
(a) 1/7 (b) 5/7 (c) 3/4 (d) 2/7

5. For which gas the ratio of specific heats (C_p/C_v) will be largest? **[JEST 2014]**
(a) mono-atomic (b) di-atomic (c) tri-atomic (d) hexa-atomic

6. Electrons of mass *m* in a thin, long wire at a temperature *T* follow a one-dimensional Maxwellian velocity distribution. The most probable speed of these electrons is, **[JEST 2015]**
(a) $\sqrt{\left(\frac{kT}{2\pi m}\right)}$ (b) $\sqrt{\left(\frac{2kT}{m}\right)}$ (c) 0 (d) $\sqrt{\left(\frac{8kT}{\pi m}\right)}$

7. The entropy temperature diagram of two Carnot engines, *A* and *B*, are shown in the figure 5. The efficiencies of the engines are η_A and η_B, respectively. Which one of the following equalities is correct ? **[JEST 2015]**

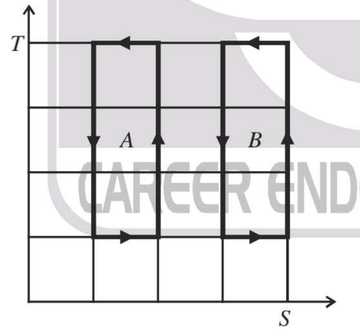


Figure 5

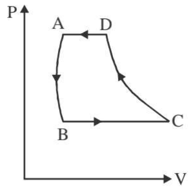
(a) $\eta_A = \eta_B/2$ (b) $\eta_A = \eta_B$ (c) $\eta_A = 3\eta_B$ (d) $\eta_A = 2\eta_B$

ANSWER KEY

1. (d)	2. (d)	3. (a)	4. (d)	5. (a)	6. (c)	7. (b)

TIFR Previous Years' Questions

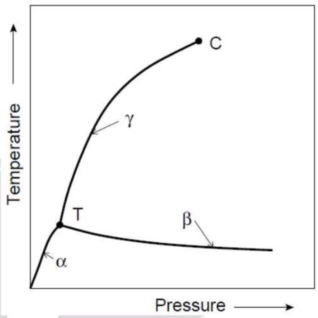
1. The pV diagram given below represents a **[TIFR 2010]**



(a) Carnot refrigerator (b) Carnot engine

(c) gas turbine refrigerator (d) gas turbine engine

2. The phase diagram of a pure substance is given in the figure below, where 'T' denotes the triple point and 'C' denotes the critical point.



The phase transitions occurring along the lines marked α, β and γ are **[TIFR 2011]**

(a) α = melting; β = condensation; γ = sublimation

(b) α = sublimation; β = vaporisation; γ = melting

(c) α = melting; β = vaporisation; γ = condensation

(d) α = sublimation; β = melting; γ = vaporisation

3. Consider the CO molecule as a system of two point particles which has both translational and rotational degrees of freedom. Using classical statistical mechanics, the molar specific heat C_V of CO gas is given in terms of the Boltzmann constant k_B by **[TIFR 2014]**

(a) $\frac{5}{2}k_B$ (b) $2k_B$ (c) $\frac{3}{2}k_B$ (d) $\frac{1}{2}k_B$

4. An ideal gas at a temperature T is enclosed in a rigid container whose walls are initially at temperature T_1, where $T_1 < T$. The walls are covered on the outside with perfect thermal insulation and the system is allowed to come to equilibrium. The pressure exerted by the gas on the walls of the container

(a) remains constant throughout **[TIFR 2014]**

(b) is lower at the initial state than at the final state.

(c) is higher at the initial stage than at the final state.

(d) is the same at the initial and final stages.

5. The equation of state for a gas is given by $\left[p+\left(\frac{\alpha N}{V}\right)^2\right](V-\beta N)=Nk_BT$ where P, V, T, N and k_B represent pressure, volume, temperature, number of atoms and the Boltzmann constant, respectively, while α and β are constants specific to the gas.

If the critical point C corresponds to a point of inflexion of the p-V curve, then the critical volume V_C and critical pressure p_c for this gas are given by **[TIFR 2016]**

(a) $V_C=3\beta N,\ p_c=\alpha^2/3\beta^2$ (b) $V_C=3\beta N,\ p_c=\alpha/27\beta^2$

(c) $V_C=3\beta N,\ p_c=8\alpha^2/27\beta$ (d) $V_C=3\beta N,\ p_c=\alpha^2/27\beta^2$

ANSWER KEY

1. (a) 2. (d) 3. (a) 4. (c) 5. (d)

Other Examinations Previous Years' Questions

1. A carnot engine operates between a heat source at 500 K and a heat sink at 300 K. The temperature of the source is increased by 20 K. In order that the efficiecy of the engine remain unchanged, the temperature of the sink should be changed by

(a) + 20 K (b) +12 K (c) –12 K (d) –20 K

2. In an infinitesimal reversible thermodynamic process, which of the following quantity depends only on the initial and the final states of the system and is independent of the path between these states?

(a) $T\Delta Q$ (b) $\Delta Q/T$ (c) $T/\Delta Q$ (d) $T^2/\Delta Q$

3. Above a temperature T_0, the specific heat of a certain gas is approximate 3.5R, and is approximately 2.5R below T_0. The gas is most likely

(a) oxygen (b) ozone (c) carbon dioxide (d) argon

4. During the isothermal reversible expansion of an ideal gas,

(a) the entropy and the pressure both increase
(b) the entropy and the pressure both decrease
(c) the entropy increases and the pressure decreases
(d) the entropy decreases and the pressure increases

5. The volume of a thermodynamic system increases irreversibly by an incremental amount δV. If P is the pressure, the work done on the system is:

(a) $\delta W = P\,\delta V$ (b) $\delta W = -P\,\delta V$ (c) $\delta W < P\,\delta V$ (d) $\delta W > -P\,\delta V$

6. Consider an elastic string of length L under tension τ at temperature T. Let U and S be its internal energy and entropy, respectively. Then the conjugate pairs of thermodynamic variables are

(a) (τ, T) and (L, S) (b) (L, τ) and (T, S)
(c) (T, L) and (τ, U) (d) (L, S) and (T, U)

7. A carnot engine can be used to define an absolute temperature scale. This is made possible by the following property of the carnot engine.

(a) It involves an adiabatic as well as an isothermal process.
(b) It is a cyclic process.
(c) It is most efficient engine operating between two given temperatures.
(d) It conserves energy.

8. Which is the most effective way of increasing the efficiency of a carnot engine operating between temperatures T_1 and $T_2 (T_1 > T_2)$?

(a) Increasing T_1, keeping T_2 fixed. (b) Increasing T_2, keeping T_1 fixed.
(c) Decreasing T_2 and increasing T_1 (d) Increasing T_2 and decreasing T_1.

9. The quantity that remains constant in Joule-Thomson expansion is

(a) $U + pV$ (b) $U + pV - TS$ (c) U (d) $U - TS$

10. A steam turbine is operated with an intake temperature 427ºC and exhaust temperature 107ºC. Heat input to the turbine is Q. Then maximum work turbine can do is:

(a) 0.5Q (b) 0.37Q (c) Q (d) 0.25 Q

11. For a system with constant volume and temperature, the thermodynamic equilibrium corresponds to the state with minimum?

(a) Helmholtz free energy (b) Gibbs thermodynamic potential
(c) Internal energy (d) Enthalpy

12. A gas of argon atoms is kept in a container of volume V and pressure P and is in equilibrium with a heat bath at temperature T. The average energy of an argon atom in the gas is:

(a) RT (b) $3k_BT$ (c) $\frac{3}{2}k_BT$ (d) 3 RT

Where k_B is Boltzman constant.

13. The molar specific heat of a gas as given from the kinetic theory is $\frac{5}{2}R$. If it is not specified whether it is C_P or C_V, one could conclude that the molecules of the gas.

(a) Are definitely monatomic (b) Are definitely rigid diatomic

(c) Are definitely non-rigid diatomic (d) Can be monatomic or rigid diatomic.

14. Isothermal compressibility κ_T of a substance is defined as $\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$. Its value for n mole of an ideal gas will be

(a) $\frac{1}{P}$ (b) $\frac{n}{P}$ (c) $-\frac{1}{P}$ (d) $-\frac{n}{P}$

15. Identify the correct statement.

(a) The entropy of a system always increases when it undergoes an irreversible process.

(b) The entropy of a system always decreases when it undergoes an irreversible process.

(c) The second law of thermodynamics follows directly from principle of conservation of energy.

(d) The internal energy of an ideal gas depends on its temperature.

16. An ideal gas undergoes an isothermal expansion (at a constant temperature T) to double its volume. The change in the entropy per mole is

(a) 2R (b) –Rln2 (c) Rln2 (d) $\frac{R}{2}$

17. One mole of a monoatomic perfect gas initially at temperature 0ºC expands from volume V_0 to $2V_0$ at constant pressure. The specific heat at constant volume of the gas is 20.8 Jmol^{-1}K^{-1}. The amount of heat absorbed is nearly

(a) 1179 J (b) 1779 J (c) 1979 J (d) 7972 J

18. When 100g ice at 0ºC melts, the change in entropy in cal/K is about (Take latent heat of fusion (melting) as 80 cal/g.)

(a) 0 (b) 0.34 (c) 29.3 (d) 39.2

19. The 10g of ice at 0ºC is converted into water at the same temperature. Take latent heat of fusion as 80 cal/g. The change in entropy in cal/K is about

(a) zero (b) 0.34 (c) 2.93 (d) 3.92

20. Consider oxygen gas at 300K having the mass of its molecule as 5.31×10^{-26} kg. The root mean square speed of its molecules, is about

(a) 284 m/s (b) 248 m/s (c) 348 m/s (d) 484 m/s

21. The power per unit area emitted by a surface of a blackbody (in Wm^{-2}) at temperature 230 K is about

(a) 112.67 (b)128.67 (c) 158.67 (d) 178.67

22. The heat capacity at costant pressure of a certain gas is 1.4 times its heat capacity at constant volume. A best model for the molecules of this gas is

(a) rigid triangles (b) point particles
(c) rigid dumbbells (d) non- rigid dumbbells that can vibrate

23. A rigid triangular molecule consists of three non-collinear atoms joined by rigid rods. The constant pressure molar specific heat (C_p) of an ideal gas consisting of such molecules is

(a) $6R$ (b) $5R$ (c) $4R$ (d) $3R$

24. An ideal gas undergoes an isothermal expansion (at a constant temperature T) from an initial volume V_1 to a final volume V_2. The change in the entropy per mole is:

(a) $R(V_1/V_2)$ (b) $R\ln|V_1-V_2|$ (c) $R\ln(V_1/V_2)$ (d) $R\ln(V_2/V_1)$

25. A gas, the molecules of which have mass m, is at equilibrium at absolute temperature T. The root-mean-square of the relative velocity between any two molecules of the gas is:

(a) $4k_BT/\pi m$ (b) $3k_BT/2m$ (c) $3k_BT/m$ (d) $6k_BT/m$

26. Let V_x, V_y and V_z denote the components of the velocity along x–,y– and z=-directions respectively, of an ideal gas particle. At the absolute temperature T, the average value of the product $V_x^2\ V_y^2\ V_z^2$ is proportional to

(a) T (b) $T^{3/2}$ (c) T^3 (d) T^6

27. The specific heat at constant volume of the electrons in a metal $C_v = \alpha T$. Where T is the temperature and α is a constant. The specific entropy s is given by

(a) κ_B (b) $\sqrt{\kappa_B \alpha T}$ (c) αT (d) $\frac{\alpha^2 T^2}{\kappa_B}$

28. In 1-dimension, an ensemble of *N* classical particles has energy of the form $E = \frac{p_x^2}{2m} + \frac{1}{2}kx^2$. The average internal energy of the system at temperature *T* is

(a) $\frac{3}{2}Nk_BT$ (b) $\frac{1}{2}Nk_BT$ (c) $3Nk_BT$ (d) Nk_BT

29. A system of weakly interacting two-dimensional harmonic oscillators is in thermal conact with a heat bath of absolute temperature T. The average kinetic energy of an oscillator is:

(a) k_BT (b) $2k_BT$ (c) $3\,k_BT/2$ (d) $k_BT/2$

ANSWER KEY

1. (b)	**2. (b)**	**3. (a)**	**4. (c)**	**5. (a)**	**6. (b)**	**7. (c)**
8. (c)	**9. (a)**	**10. (a)**	**11. (a)**	**12. (c)**	**13. (d)**	**14. (a)**
15. (d)	**16. (c)**	**17. (d)**	**18. (c)**	**19. (c)**	**20. (d)**	**21. (c)**
22. (c)	**23. (c)**	**24. (d)**	**25. (d)**	**26. (c)**	**27. (c)**	**28. (d)**
29. (a)						

GATE Previous Years' Questions

1. Choose the correct alternatives **[GATE 1999]**
(a) internal energy of a Van der Waals gas at a given temperature increases as the volume increases
(b) internal energy of a perfect gas at a given temperature increases as the volume increases
(c) internal energy of a Fermi gas at a given temperature increases as the volume increases
(d) internal energy of a Fermi gas at a given temperature decreases as the volume increases.

2. A Carnot cycle operates on a working substance between two reservoirs at temperature T_1 and T_2, with $T_1 > T_2$. During each cycle, an amount of heat Q_1 is extracted from the reservoir at T_1 and an amount Q_2 is delivered to the reservoir at T_2. Which of the following statements is INCORRECT? **[GATE 2011]**

(a) Work done in one cycle is $Q_1 - Q_2$. (b) $\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$

(c) Entropy of the hotter reservoir decreases
(d) Entropy of the universe (cosisting of the working substance and the two reservoirs) increases.

3. Water freezes at 0ºC at atmospheric pressure $(1.01\times10^5 \text{ Pa})$. The densities of water and ice at this temperature and pressure are 1000 kg/m^3 and 934 kg/m^3 respectively. The latent heat of fusion is 3.34×10^5 J/kg. The pressure required for depressing the melting temperature of ice by 10ºC is ____ GPa. (up to two decimal places) **[GATE 2017]**

4. An air-conditioner maintains the room temperature at 27 ºC while the outside temperature is 47 ºC. The heat conducted through the walls of the room from outside to inside due to temperature difference is 7000 W. The minimum work done by the compressor of the air-conditioner per unit time is ________ W. **[GATE 2018]**

5. Consider a one-dimensional gas of N non-interacting particles of mass m with the Hamiltonian of a single particle given by, **[GATE 2019]**

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2(x^2 + 2x)$$

The high temperature specific heat in units of $R = Nk_B$ (k_B is the Boltzmann constant) is
(a) 1 (b) 1.5 (c) 2 (d) 2.5

6. In a thermally insulated container, 0.01 kg of ice at 273 K is mixed with 0.1 kg of water at 300 K. Neglecting the specific heat of the container, the change in the entropy of the system in J/K on attaining thermal equilibrium (rounded off to two decimal places) is __________. **[GATE 2019]**

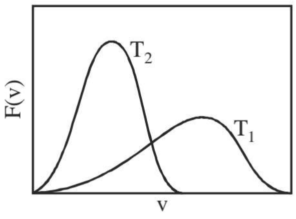
(Specific heat of water is 4.2 kJ/kg-K and the latent heat of ice is 335 kJ/kg).

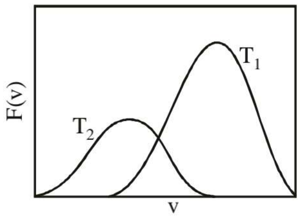
ANSWER KEY

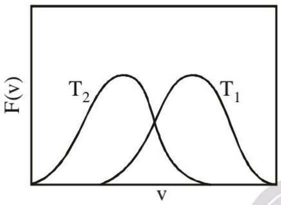
1. (a,d) 2. (d) 3. (0.15 to 0.19) 4. (466 to 467) 5. (a)
6. (1.02 to 1.04)

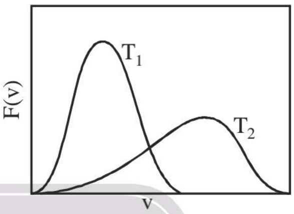
CSIR-UGC-NET Previous Years' Questions

1. For temperature $T_1 > T_2$, the qualitative temperature dependence of the probability distribution F(v) of the speed v of a molecule in three dimensions is correctly represented by the following figure: **[CSIR June 2013]**

(a)


(b)


(c)


(d)


2. A system of N classical non-interacting particles, each of mass *m*, is at a temperature T and is confined by the external potential $V(r) = \frac{1}{2}Ar^2$ (where A is a constant) in three dimensions. The internal energy of the system is **[CSIR June 2013]**

(a) $3Nk_BT$ (b) $\frac{3}{2}Nk_BT$ (c) $N(2mA)^{3/2}k_BT$ (d) $N\sqrt{\frac{A}{m}}\ln\left(\frac{k_BT}{m}\right)$

3. Ten grams of ice at 0°C is added to a beaker containing 30 grams of water at 25°C. What is the final temperature of the system when it comes to thermal equilibrium? (The specific heat of water is 1cal/gm/°C and latent heat of melting of ice is 80 cal/gm) **[NET June 2013]**

(a) 0°C (b) 7.5°C (c) 12.5°C (d) –1.25°C

4. A vessel has two compartments of volume V_1 and V_2, containing an ideal gas at pressures p_1 and p_2, and temperatures T_1 and T_2 respectively. If the wall separating the compartments is removed, the resulting equilibrium temperature will be **[NET June 2013]**

(a) $\frac{p_1T_1 + p_2T_2}{p_1 + p_2}$ (b) $\frac{V_1T_1 + V_2T_2}{V_1 + V_2}$

(c) $\frac{p_1V_1 + p_2V_2}{(p_1V_1/T_1) + (p_2V_2/T_2)}$ (d) $(T_1T_2)^{1/2}$

5. The speed v of the molecules of mass *m* of an ideal gas obeys Maxwell's velocity distribution law at an equilibrium temperature T. Let (v_x, v_y, v_z) denote the components of the velocity and k_B the Boltzmann constant. The average value of $(\alpha v_x - \beta v_y)^2$, where α and β are constants, is **[CSIR Dec. 2013]**

(a) $(\alpha^2 - \beta^2)k_BT/m$ (b) $(\alpha^2 + \beta^2)k_BT/m$ (c) $(\alpha + \beta)^2 k_BT/m$ (d) $(\alpha - \beta)^2 k_BT/m$

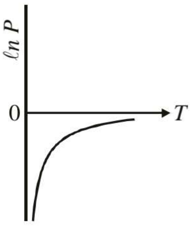
6. A Carnot cycle operates as a heat engine between two bodies of equal heat capacity until their temperatures become equal. If the initial temperatures of the bodies are T_1 and T_2, respectively, and $T_1 > T_2$ then their common final temperature is **[CSIR Dec. 2013]**

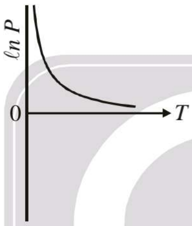
(a) T_1^2/T_2 (b) T_2^2/T_1 (c) $\sqrt{T_1T_2}$ (d) $\frac{1}{2}(T_1+T_2)$

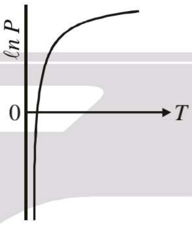
7. When a gas expands adiabatically from volume V_1 to V_2 by a quasi-static reversible process, it cools from temperature T_1 to T_2. If now the same process is carried out adiabatically and irreversibly, and T_2' is the temperature of the gas when it has equilibrated, then **[NET Dec. 2014]**

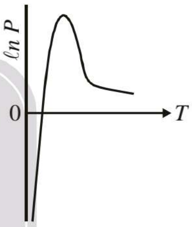
(a) $T_2' = T_2$ (b) $T_2' > T_2$ (c) $T_2' = T_2\left(\frac{V_2 - V_1}{V_2}\right)$ (d) $T_2' = \frac{T_2V_1}{V_2}$

8. The condition for the liquid and vapour phases of a fluid to be in equilibrium is given by the approximate equation $\frac{dP}{dT} \approx \frac{Q_\ell}{Tv_{\text{vap}}}$ (Clausius-Clayperon equation), where v_{vap} is the volume per particle in the vapour phase, and Q_l is the latent heat, which may be taken to be a constant. If the vapour obeys ideal gas law, which of the following plots is correct? **[NET June 2015]**

(a)


(b)


(c)


(d)


9. The heat capacity of (the interior of) a refrigerator is 4.2 kJ/K. The minimum work that must be done to lower the internal temperature from 18℃ to 17℃ when the outside temperature is 27℃ will be **[NET Dec. 2015]**

(a) 2.20 kJ (b) 0.80 kJ (c) 0.30 kJ (d) 0.14 kJ

10. A silica particle of radius 0.1 μm is put in a container of water at $T = 300$ K. The densities of silica and water are 2000 kg/m³ and 1000 kg/m³, respectively. Due to thermal fluctuations, the particle is not always at the bottom of the container. The average height of the particle above the base of the container is approximately **[NET Dec. 2016]**

(a) 10^{-3} m (b) 3×10^{-4} m (c) 10^{-4} m (d) 5×10^{-5} m

11. A box, separated by a movable wall, has two compartment filled by a monoatomic gas of $\frac{C_P}{C_V} = \gamma$. Initially the volumes of the two compartments are equal, but the pressure are $3P_0$ and P_0, respectively. When the wall is allowed to move, the final pressure in the two compartments become equal. The final pressure is **[NET June 2017]**

(a) $\left(\frac{2}{3}\right)^{\gamma} P_0$ (b) $3\left(\frac{2}{3}\right)^{\gamma} P_0$ (c) $\frac{1}{2}(1+3^{1/\gamma})^{\gamma} P_0$ (d) $\left(\frac{3^{1/\gamma}}{1+3^{1/\gamma}}\right)^{\gamma} P_0$

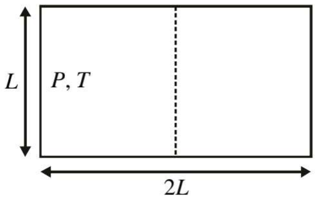
12. The pressure P of a system of N particles contained in a volume V at a temperature T is given by

$$P = nk_BT - \frac{1}{2}an^2 + \frac{1}{6}bn^3$$

where n is the number density and a and b are temperature independent constants. If the system exhibits a gas-liquid transition, the critical temperature is **[NET June 2018]**

(a) $\frac{a}{bk_B}$ (b) $\frac{a}{2b^2k_B}$ (c) $\frac{a^2}{2bk_B}$ (d) $\frac{a^2}{b^2k_B}$

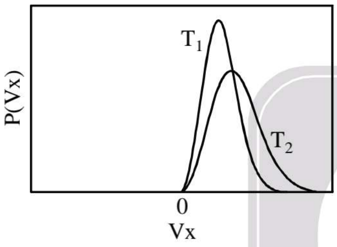
13. A thermally insulated chamber of dimensions $(L, L, 2L)$ is partitioned in the middle. One side of the chamber is filled with n moles of an ideal gas at a pressure P and temperature T, while the other side is empty. At $t = 0$, the partition is removed and the gas is allowed to expand freely. The time to reach equilibrium varies as

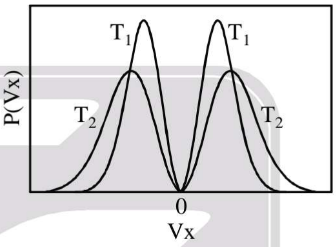


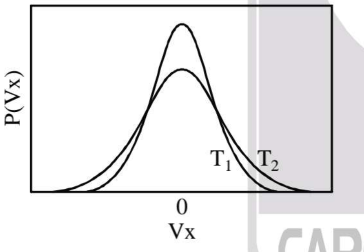
[NET June 2018]

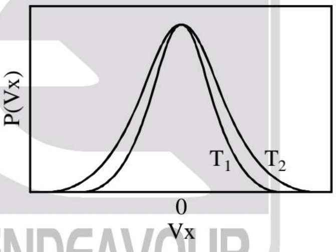
(a) $n^{1/3}L^{-1}T^{1/2}$ (b) $n^{2/3}LT^{-1/2}$ (c) $n^{0}LT^{-1/2}$ (d) $nL^{-1}T^{1/2}$

14. Consider a classical gas in thermal equilibrium at temperatures T_1 and T_2. Which of the following graphs correctly represents the qualitative behaviour of the probability density function of the x-component of the velocity? **[NET June 2019]**

(a)


(b)


(c)


(d)


15. The Hamiltonian of a classical nonlinear one dimensional oscillator is $H = \frac{1}{2m}p^2 + \lambda x^4$, where $\lambda > 0$ is a constant. The specific heat of a collection of N independent such oscillators is **[NET June 2019]**

(a) $\frac{3Nk_B}{2}$ (b) $\frac{3Nk_B}{4}$ (c) Nk_B (d) $\frac{Nk_B}{2}$

16. A mole of gas at initial temperature T_i comes into contact with a heat reservoir at temperature T_f and the system is allowed to reach equilibrium at constant volume. If the specific heat of the gas is $C_V = \alpha T$, where α is a constant, the total change in entropy is **[NET Dec. 2019]**

(a) Zero

(b) $\alpha\left(T_f - T_i\right) + \frac{\alpha}{2T_f}\left(T_f - T_i\right)^2$

(c) $\alpha\left(T_f - T_i\right)$

(d) $\alpha\left(T_f - T_i\right) + \frac{\alpha}{2T_f}\left(T_f^2 - T_i^2\right)$

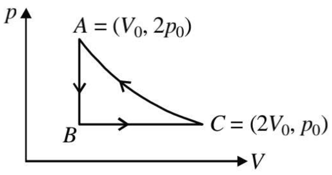
17. The pressure p of a gas depends on the number density ρ of particles and the temperature T as

$$p = k_B T\rho - B_2\rho^2 + B_3\rho^3, \text{ where } B_2 \text{ and } B_3 \text{ are positive constants.}$$

Let T_c, ρ_c and p_c denote the critical temperature, critical number density and critical pressure, respectively. The ratio $\rho_c k_B T_c / p_c$ is equal to

(a) 1/3 (b) 3 (c) 8/3 (d) 4 **[NET Dec. 2019]**

18. One mole of an ideal gas is taken through the cycle ABC once in the pV-diagram, as shown in the figure. The segment BC is horizontal, while AB is vertical. The coordinates of A and C are $(V_0, 2p_0)$ and $(2V_0, p_0)$ respectively, and they are joined by an isotherm. The total work done is **[NET Dec. 2019]**



(a) $p_0V_0(2\ln 2 - 1)$ (b) $p_0V_0 \ln 2$ (c) $2p_0V_0 \ln 2$ (d) $p_0V_0(1 - \ln 2)$

19. A box of length L is divided into two parts by a movable wall. Initially the left part of the box contains only Kr gas and the right contains only He gas, both at the same pressure and temperature. The material of the wall is semi-permeable that allows He atoms to pass through, but not Kr atoms. Assuming that the gases are ideal, the position of the wall (with respect to the left of the box) after reaching equilibrium, will be at

(a) $L/2$ (b) $2L/3$ (c) $3L/4$ (d) L **[NET Dec. 2019]**

ANSWER KEY

1. (a)	2. (a)	3. (a)	4. (c)	5. (b)	6. (c)	7. (b)
8. (c)	9. (d)	10. (c)	11. (c)	12. (c)	13. (c)	14. (c)
15. (b)	16. (d)	17. (b)	18. (a)	19. (d)		

JEST Previous Years' Questions

1. Consider an ideal gas of mass m at temperature T_1 which is mixed isobarically (i.e., at constant pressure) with an equal, mass of same gas at temperature T_2 in a thermally insulated container. What is the change of entropy of the universe ? **[JEST 2012]**

(a) $2mC_p \,\text{In}\left(\dfrac{T_1+T_2}{2\sqrt{T_1T_2}}\right)$ (b) $2mC_p \,\text{In}\left(\dfrac{T_1-T_2}{2\sqrt{T_1T_2}}\right)$

(c) $2mC_p \,\text{In}\left(\dfrac{T_1+T_2}{2T_1T_2}\right)$ (d) $2mC_p \,\text{In}\left(\dfrac{T_1-T_2}{2\sqrt{T_1T_2}}\right)$

2. An ideal gas is compressed adiabatically from an initial volume V to a final volume αV and a work W is done on the system in doing so. The final pressure of the gas will be $\left(\gamma = \dfrac{C_P}{C_V}\right)$, **[JEST 2015]**

(a) $\dfrac{W}{V^\gamma}\dfrac{1-\gamma}{\alpha-\alpha^\gamma}$ (b) $\dfrac{W}{V^\gamma}\dfrac{\gamma-1}{\alpha-\alpha^\gamma}$ (c) $\dfrac{W}{V}\dfrac{1-\gamma}{\alpha-\alpha^\gamma}$ (d) $\dfrac{W}{V}\dfrac{\gamma-1}{\alpha-\alpha^\gamma}$

3. An ideal gas with adiabatic exponent γ undergoes a process in which its pressure P is related to its volume V by the relation $P = P_0 - \alpha V$, where P_0 and α are positive constants. The volume starts from being very close to zero and increases monotonically to P_0/α. At what value of the volume during the process does the gas have maximum entropy? **[JEST 2016]**

(a) $\dfrac{P_0}{\alpha(1+\gamma)}$ (b) $\dfrac{\gamma P_0}{\alpha(1-\gamma)}$ (c) $\dfrac{\gamma P_0}{\alpha(1+\gamma)}$ (d) $\dfrac{P_0}{\alpha(1-\gamma)}$

4. An ideal gas has a specific heat ratio $C_P/C_V = 2$. Starting at a temperature T_1 the gas under goes an isothermal compression to increase its density by a factor of two. After this an adiabatic compression increases its pressure by a factor of two. The temperature of the gas at the end of the second process would be:

(a) $\dfrac{T_1}{2}$ (b) $\sqrt{2}T_1$ (c) $2T_1$ (d) $\dfrac{T_1}{\sqrt{2}}$ **[JEST 2016]**

5. After the detonation of an atom bomb, the spherical ball of gas was found to be 15 meter radius at a temperature of 3×10^5 K. Given the adiabatic expansion coefficient $\gamma = \frac{5}{3}$, what will be the radius of the ball when its temperature reduces to 3×10^3 K ? **[JEST 2017]**

(a) 156 m (b) 50 m (c) 150 m (d) 100 m

6. If the Hamiltonian of a classical particle is $H = \dfrac{p_x^2 + p_y^2}{2m} + xy$, then $\left\langle x^2 + xy + y^2 \right\rangle$ at temperature T is equal to **[JEST 2017]**

(a) k_BT (b) $\dfrac{1}{2}k_BT$ (c) $2k_BT$ (d) $\dfrac{3}{2}k_BT$

7. Let a particle of mass 1×10^{-9} kg, constrained to have one dimensional motion, be initially at the origin $(x = 0\,\text{m})$. The particle is in equilibrium with a thermal bath $(k_B T = 10^{-8}\,\text{J})$. What is $\langle x^2 \rangle$ of the particle after a time $t = 5\,\text{s}$? **[JEST 2017]**

8. For a classical system of non-interacting particles in the presence of a spherically symmetric potential $V(r) = \gamma r^3$, what is the mean energy per particle? γ is a constant. **[JEST 2018]**

(a) $\frac{3}{2}k_B T$ (b) $\frac{5}{2}k_B T$ (c) $\frac{3}{2}\gamma k_B T$ (d) $\frac{3}{2}\gamma k_B T$

9. In a thermodynamic process, the volume of one mole of an ideal is varied as $V = aT^{-1}$, where a is a constant. The adiabatic exponent of the gas is γ. What is the amount of heat received by the gas if the temperature of the gas increases ΔT in the process? **[JEST 2018]**

(a) $R\Delta T$ (b) $\frac{R\Delta T}{1-\gamma}$ (c) $\frac{R\Delta T}{2-\gamma}$ (d) $R\Delta T\frac{2-\gamma}{\gamma-1}$

10. A theoretical model for a real (non-ideal) gas gives the following expressions for the internal energy (U) and the pressure (P),

$$U(T,V) = aV^{-2/3} + bV^{2/3}T^2 \quad \text{and} \quad P(T,V) = \frac{2}{3}aV^{-5/3} + \frac{2}{3}bV^{-1/3}T^2$$

where a and b are constants. Let V_0 and T_0 be the initial volume and initial temperature respectively. If the gas expands adiabatically, the volume of thegas is proportional to **[JEST 2018]**

(a) T (b) $T^{3/2}$ (c) $T^{-3/2}$ (d) T^{-2}

11. A large cylinder of radius R filled with particles of mas m. The cylinder spins about its axis at an angular speed ω radians per second, providing an acceleration g for the particle at the rim. If the temperature T is constant insider the cylinder, what is the ratio of air pressure P_0 at the axis to the pressure P_c at the rim?

(a) $\exp\left[\frac{mgR}{2k_bT}\right]$ (b) $\exp\left[-\frac{mgR}{2k_bT}\right]$ (c) $\frac{mgR}{2k_bT}$ (d) $\frac{2k_bT}{mgR}$ **[JEST 2018]**

12. An ideal fluid is subjected to a thermodynamic process described by $\rho = CV^{-\alpha}$ and $P = n\rho^{\Gamma}$, where ρ is energy density and P is pressure. For what values of n and Γ, the process is adiabatic if the volume is changed slowly? **[JEST 2018]**

(a) $\Gamma = \alpha - 1, n = 1$ (b) $\Gamma = 1 - \alpha, n = \alpha$ (c) $\Gamma = 1, n = \alpha - 1$ (d) $\Gamma = \alpha, n = 1 - \alpha$

13. In an experiment, certain quantity of an ideal gas at temperature T_0, pressure P_0 and volume V_0 is heated by a current flowing through a wire for a duration of t seconds. The volume is kept constant and the pressure changes to P_1. If the experiment is performed at constant pressure starting with the same initial conditions, the volume changes from V_0 to V_1. The ratio of the specific heats at constant pressure and constant volume is

(a) $\frac{P_1 - P_0}{V_1 - V_0}\frac{V_0}{P_0}$ (b) $\frac{P_1 - P_0}{V_1 - V_0}\frac{V_1}{P_1}$ (c) $\frac{P_1V_1}{P_0V_0}$ (d) $\frac{P_0V_0}{P_1V_1}$ **[JEST 2018]**

14. A diatomic ideal gas at room temperature, is expanded at a constant pressure P_0. If the heat absorbed by the gas is $Q = 14$ Joules, what is the maximum work in Joules that can be extracted from the system? **[JEST 2019]**

ANSWER KEY

1. (a)	2. (d)	3. (c)	4. (b)	5. (c)	6. (a)	7. (250)
8. (b)	9. (d)	10. (c)	11. (b)	12. (c)	13. (a)	14. (4)

TIFR Previous Years' Questions

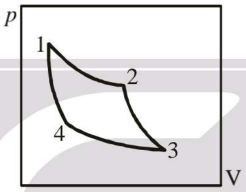
1. A car tyre is slowly pumped up to a pressure of 2 atmospheres in an environment at 15°C. At this point, it bursts. Assuming the sudden expansion of the air (a mixture of O_2 and N_2) that was inside the tyre to be adiabatic, its temperature after the burst is **[TIFR 2010]**

(a) –55°C (b) –37°C (c) –26°C (d) +9°C

2. A classical ideal gas, consisting of N particles ($N \to \infty$) is confined in a box of volume V at temperature T and pressure p. The probability that, at any instant of time, a small sub-volume v_0 becomes totally void (i.e. no particles inside), due to a spontaneous statistical fluctuation, is **[TIFR 2013]**

(a) $\exp\left(\frac{-v_0}{V}\right)$ (b) $\exp\left(\frac{-Nv_0}{V}\right)$ (c) $\frac{v_0}{V}\exp\left(\frac{-pV}{NT}\right)$ (d) $\frac{pv_0}{NT}$

3. The PV-diagram for a Carnot cycle executed by an ideal gas with $C_P/C_V = \gamma > 1$ is shown below. Note that 1, 2, 3 and 4 label the change-over points in the cycle. If, for this cycle, $\frac{T_2}{T_3} = \left(\frac{p_2}{p_3}\right)^X$, then $X =$



(a) $1-\frac{1}{\gamma}$ (b) 0 (c) 1 (d) $-\frac{1}{\gamma}$ **[TIFR 2013]**

4. The entropy S of a black hole is known to be of the form **[TIFR 2013]**

$$S = \alpha k_B A$$

where A is the surface area of the black hole and α is a constant, which can be written in terms of c (velocity of light in vacuum), $\hbar$ (reduced Planck's constant) and G_N (Newton's constant of gravitation). Taking the radius of the black hole as $R = \frac{2G_N M}{c^2}$ it follows that the entropy S is [λ is a numerical constant]

(a) $\frac{G_N^2 M^2 k_B}{\lambda(\hbar c)^4}$ (b) $\frac{\hbar c k_B}{\lambda G_N M}$ (c) $\frac{G_N^2 M^2 k_B}{\lambda \hbar c^4}$ (d) $\frac{G_N M^2 k_B}{\lambda \hbar c}$

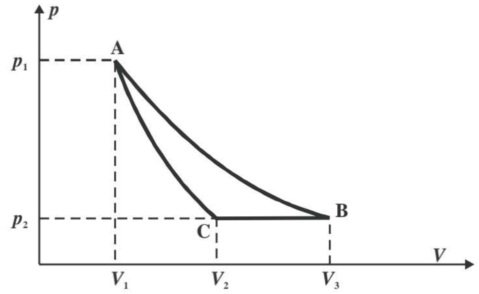
5. A monatomic gas is described by the equation of state $p(V - bn) = nRT$ where b and R are constants and other quantities have their usual meanings. The maximum density (in moles per unit volume) to which this gas can be compressed is **[TIFR 2013]**

(a) $\frac{1}{bn}$ (b) b (c) $\frac{1}{b}$ (d) infinity

6. A certain amount of fluid with heat capacity C_F Joules/°C is initially at a temperature 0° C. It is then brought into contact with a heat bath at a temperature of 100° C, and the system is allowed to come into equilibrium. In this process, the entropy (in Joules / °C) of the Universe changes by **[TIFR 2013]**

(a) $100C_F$ (b) 0 (c) $0.055C_F$ (d) $0.044C_F$

7. One mole of an ideal gas undergoes the cycle ACBA shown in the PV diagram below.



One of the curved lines in the cycle represents an isothermal change at temperature T, while the other represents an adiabatic change. The net heat gained by the gas in this cycle is **[TIFR 2014]**

(a) $-p_2(V_3-V_2)+RT\ln\frac{V_2}{V_1}$ (b) $-p_2(V_3-V_2)+RT\ln\frac{V_3}{V_1}$

(c) $-p_2(V_3-V_2)+\gamma RT\left(V_2^{1-\gamma}-V_1^{1-\gamma}\right)$ (d) $(p_1V_1-p_2V_2)-RT\ln\frac{V_3}{V_1}$

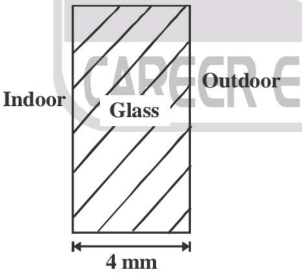
8. A thermally-insulated container of volume V_0 is divided into two equal halves by a non-permeable partition. A real gas with equation of state. **[TIFR 2014]**

$$b^3\left(p+\frac{a^2}{V^3}\right)=nRT$$

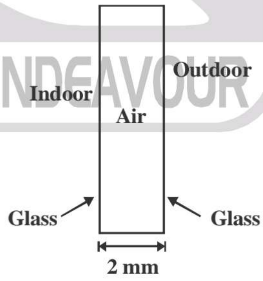
where 'a' and 'b' are constants, is confined to one of these halves at a temperature T_0. The partition is now removed suddenly and the gas is allowed to expand to fill the entire container. The final temperature of the gas, in terms of its specific heat C_V, will be

(a) $T_0-\frac{3a^2}{2C_VV_0^2}$ (b) $T_0-\frac{2a^2}{3C_VV_0^2}$ (c) $T_0+\frac{3a^2}{2C_VV_0^2}$ (d) $T_0+\frac{2a^2}{3C_VV_0^2}$

9. A manufacturer is able to offer two models of heat-conserving windows, as described below



Window A



Window B

Window A is a simple pane of glass, 4 mm thick. Window B, on the other hand, consists of two extremely thin panes of glass, separated by an air gap of 2mm, as shown in the figure below above. If the thermal conductivity of glass is known to be 0.8 $Wm^{-1}K^{-1}$ and that of air is 0.025 $Wm^{-1}K^{-1}$, then the ratio of heat flow Q_A through window A to the heat flow Q_B through Window B is given by

$$\frac{Q_A}{Q_B}=$$ **[TIFR 2014]**

(a) $\frac{1}{16}$ (b) $\frac{1}{4}$ (c) 4 (d) 16

10. In a cold country, in winter, a lake was freezing slowly. It was observed that it took 2 hours to form a layer of ice 2 cm thick on the water surface. Assuming a constant thermal conductivity throughout the layer, the thickness of ice would get doubled after **[TIFR 2015]**

(a) 2 more hours (b) 4 more hours (c) 6 more hours (d) 8 more hours

11. The equation of state of a gas is given by **[TIFR 2015]**

$$V = \frac{RT}{P} - \frac{b}{T}$$

where R is the gas constant and b is another constant parameter. The specific heat at constant pressure C_P and the specific heat at constant volume C_V for this gas is related by $C_P - C_V =$

(a) R (b) $R\left(1+\frac{RT^2}{bP}\right)^2$ (c) $R\left(1+\frac{bP}{RT^2}\right)^2$ (d) $R\left(1-\frac{bP}{RT^2}\right)^2$

12. An ideal diatomic gas is initially at a temperature $T = 0°\text{C}$. Then it expands reversibly and adiabatically to 5 times its volume. Its final temperature will be approximately **[TIFR 2015]**

(a) –180°C (b) –150°C (c) –130°C (d) 0°C

13. In the temperature range 100 – 1000 C, the molar specific heat of a metal varies with temperature T (measured in degrees Celsius) according to the formula $C_p = (1 + T/5)$ J-deg C^{-1} mol^{-1}. If 0.2 kg of the metal at 600 C is brought in thermal contact with 0.1 kg of the same metal at 300 C, the final equilibrium temperature, in deg C, will be **[TIFR 2016]**

[Assume that no heat is lost due to radiation and/or other effects].

(a) 466 (b) 567 (c) 383 (d) 519

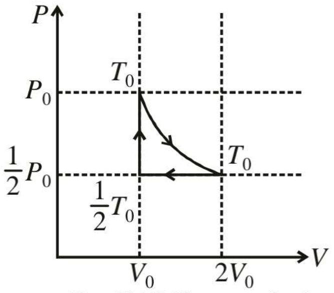
14. A thermally isolated container stores N_2 gas at 27.24 ℃ at one atmospheric pressure. Suddenly the pressure of the gas is increased to two atmospheric pressures. Assuming N_2 to behave as an ideal gas, estimate the change in temperature of the gas, in Celsius degree (℃). **[TIFR 2017]**

15. One mole of monoatomic ideal gas is initially at pressure P_0 and volume V_0. The gas then undergoes a three-stage cycle consisting of the following processes: **[TIFR 2017]**

(i) An isothermal expansion till it reaches volume $2V_0$, and heat Q flows into the gas

(ii) An isobaric compression back to the original volume V_0

(iii) An isochoric increase in pressure till the original pressure P_0 is regained



The corresponding P-V diagram is shown above

The efficiency of this cycle can be expressed as

(a) $\varepsilon = \frac{4Q+2RT_0}{4Q+RT_0}$ (b) $\varepsilon = \frac{4Q+2RT_0}{4Q-3RT_0}$ (c) $\varepsilon = \frac{4Q-2RT_0}{4Q+RT_0}$ (d) $\varepsilon = \frac{4Q-2RT_0}{4Q+3RT_0}$

16. A classical ideal gas of atoms with masses m is confined in a three-dimensional potential

$$V(x, y, z) = \frac{\lambda}{2}(x^2 + y^2 + z^2)$$

at a temperature T. If k_B is the Boltzmann constant, the root mean square (r.m.s.) distance of the atoms from the origin is **[TIFR 2018]**

(a) $\left(\frac{3k_BT}{\lambda}\right)^{1/2}$ (b) $\left(\frac{3k_BT}{2\lambda}\right)^{1/2}$ (c) $\left(\frac{2k_BT}{3\lambda}\right)^{1/2}$ (d) $\left(\frac{k_BT}{\lambda}\right)^{1/2}$

17. A heat engine is operated between two bodies that are kept at constant pressure. The constant pressure heat capacity C_p of the reservoirs is independent of temperature. Initially the reservoirs are at temperature 300 K and 402 K. If, after some time, they come to a common final temperature T_f, the process remaining adiabatic, what is the value of T_f (in Kelvin) ? **[TIFR 2018]**

18. A many-body system undergoes a phase transition between two phases A and B at a temperature T_c. The temperature-dependent specific heat at constant volume C_V of the two phases are given by $C_V^{(A)} = aT^3 + bT$ and $C_V^{(B)} = cT^3$. Assuming negligible volume change of the system, and no latent heat generated in the phase transition, T_c is **[TIFR 2018]**

(a) $\sqrt{\frac{2b}{c}}$ (b) $\sqrt{\frac{b}{c-a}}$ (c) $\sqrt{\frac{3b}{c-a}}$ (d) $\sqrt{\frac{4b}{c-a}}$

19. The molar equation of state of a gas at temperature T, pressure P and volume V is given by

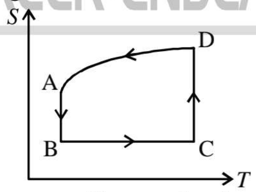
$$P = \frac{RT}{V-b} - \frac{a}{TV^2}$$

where a and b are two constants and R is the gas constant. The critical temperature and pressure for this gas will be **[TIFR 2019]**

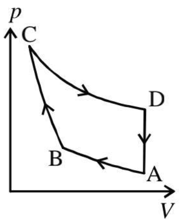
(a) $T_c = \sqrt{\frac{8a}{3Rb}},\ P_c = \frac{RT_c}{8b}$ (b) $T_c = \sqrt{\frac{a}{27Rb}},\ P_c = \frac{RT_c}{b}$

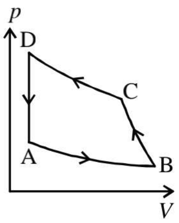
(c) $T_c = \sqrt{\frac{4a}{27Rb}},\ P_c = \frac{RT_c}{4b}$ (d) $T_c = \sqrt{\frac{8a}{27Rb}},\ P_c = \frac{RT_c}{8b}$

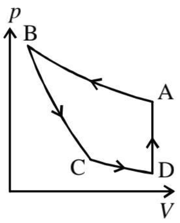
20. An ideal gas engine is run according to the shown in the s-T diagram below, where the process from D to A is known to be isochoric (i.e., maintaining V = constant).

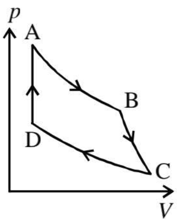


The corresponding cycle in the p-V diagram will most closely resemble **[TIFR 2019]**

(a)


(b)


(c)


(d)


21. A thermally-insulated coffee mug contains 500 g of warm coffee at 80 ℃. Assuming that the heat capacity of this liquid is 1 cal g^{-1} $℃^{-1}$ and the latent heat of fusion for ice is 80 cal g^{-1}, the amount of ice that must be dropped into the cup to convert it into cold coffee at 5 ℃ is about **[TIFR 2019]**

(a) 441 g (b) 421 g (c) 469 g (d) 471 g

ANSWER KEY

1. (c)	**2. (b)**	**3. (a)**	**4. (d)**	**5. (c)**	**6. (d)**	**7. (b)**
8. (a)	**9. (d)**	**10. (c)**	**11. (c)**	**12. (c)**	**13. (d)**	**14. (066)**
15. (d)	**16. (a)**	**17. (347)**	**18. (c)**	**19. (d)**	**20. (a)**	**21. (a)**

Other Examinations Previous Years' Questions

1. An ideal gas engine produces 1 kJ of useful work. If it operators between 400K and 300K, the amount of energy lost as waste heat is
(a) 4 kJ (b) 2kJ (c) 1kJ (d) 3kJ

2. Two insulated containers connected with a stop cock are initially evaluated. An ideal gas is filled in one of them and then allowed to expand into the other container by opening the cock. In this process the work done (W), the heat evolved (Q) and the change in internal energy (ΔU) are given by
(a) $W = 0, Q = 0$ and $\Delta U = 0$ (b) $W = +ve, Q = 0$ and $\Delta U = -ve$
(c) $W = -ve, Q = 0$ and $\Delta U = -ve$ (d) $W = -ve, Q = 0$ and $\Delta U = +ve$

3. A simple calculation for the entropy S of an ideal gas gives $S = \frac{3}{2} Nk_B \ln V + S_0$ where N is the number of molecules in the gas, V is its volume, and S_0 an arbitrary constant. This expression is erroneous because
(a) It does not contain temperature T
(b) It does not contain pressure P
(c) It is not extensive
(d) It does not take into account the interaction between the molecules of the gas.

4. N_1 molecules of oxygen are confined in volume V_1 at temperature T and have entropy S_1 ; N_2 molecules of oxygen are confined in volume V_2 at temperature T and have entropy S_2. The gases are mixed in a volume $V = V_1 + V_2$. The entropy S of the mixture is:
(a) $S < S_1 + S_2$ (b) $S = S_1 + S_2$ (c) $S > S_1 + S_2$ (d) $RT\left(\ln \frac{V_1}{V_1+V_2} - \ln \frac{V_2}{V_1+V_2}\right)$

5. Consider a perfect gas in a vessel rushing into an evacuated vessel and let the whole system be isolated. Let V_1 and V_2 denote the initial and final volumes of the gas. The increase in entropy of the system is:
(a) Zero (b) $R \ln \frac{V_2}{V_1}$ (c) $\frac{1}{T}\int_{V_1}^{V_2} du + R \ln \frac{V_2}{V_1}$ (d) $\frac{1}{T}\int_{V_1}^{V_2} du$

6. In order to attain thermal equilibrium at a fixed volume, the system minimises,
(a) S (b) $H = E + pV$ (c) $G = U - TS + pV$ (d) $F = U - TS$
Where the symbols have their usual meaning.

7. The particle density fluctuations in a pure substance are maximum at
(a) Its freezing point (b) Its boiling point (c) At critical point (d) At triple point.

8. A thermally insulated cubical box has two chambers of equal volume. Initially, one mole of a mono atomic ideal gas is placed in one of the chambers while the other chamber is kept empty. The gas slowly leaks through a small hole and eventually occupies the whole box. The change in entropy of the gas is given by
(a) $R/\ln 2$ (b) $R \ln 2$ (c) zero (d) 2R

9. One mole of ideal monoatomic gas is kept in a volume V and is at temperature T. If the volume and temperature of the gas are respectively changed to V' and T'. in such a way that the entropy of the gas is unchagned, then which of the following is true?
(a) $V' = 2V, \quad T' = T/2$ (b) $V' = 8V, \quad T' = T/4$
(c) $V' = 4V, \quad T' = 8T$ (d) $V' = V/2, \quad T' = 2T$

10. A carnot engine operates between a source at temperature 300K and a sink at temperature 200 K. Let η be the efficiency of the Carnot engine when the temperatures of both the source and the sink are decreased by the same amount. Then,

(a) $\eta < 1/3$ (b) $\eta > 1/3$ (c) $\eta = 1/3$ (d) $\eta = 1/2$

11. Consider the Maxwell relation $\left(\frac{\partial S}{\partial V}\right)_{T,N} = \left(\frac{\partial P}{\partial T}\right)_{V,N}$ where the symbols have the usual meaning. this relation can be derived from

(a) Helmholtz free energy. $F = U - TS$
(b) Gibbs free energy : $G = U - TS + PV$
(c) Grand thermodynamic potential $Z = U - TS - \mu N$
(d) Enthaply $H = U + PV$

12. Consider a sample of argon gas contained in a box of volume V. The root-mean square speed of the argon atoms is v_m. If the gas is allowed to expand at constant pressure to a volume of 2V, the root mean square speed of the gas molecules would be

(a) $\upsilon_m/2$ (b) $\sqrt{2}\,\upsilon_m$ (c) $\upsilon_m/\sqrt{2}$ (d) $2\upsilon_m$

13. A classical ideal gas in an enclosure is in equilibrium with a heat bath at a constant temperature. The ratio of the root-mean square speed V_{rms}, mean speed υ_{mean} and the most probable speed υ_{mp} namely $V_{rms} : V_{mean} : V_{mp}$ is:

(a) $\sqrt{3} : \sqrt{\frac{8}{\pi}} : \sqrt{2}$ (b) $\frac{\sqrt{3}}{2} : \sqrt{\frac{2}{\pi}} : 1$ (c) $\sqrt{\pi} : \sqrt{3} : \sqrt{2}$ (d) $\sqrt{\frac{\pi}{3}} : \sqrt{\frac{\pi}{8}} : \frac{1}{\sqrt{2}}$

14. The average speed of molecules in a gas at $20°C$ is v At what temperature it will be 2v?

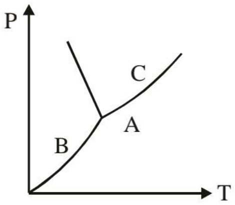
(a) 899°C (b) 586°C (c) 80°C (d) 40°C

15. At 100ºC, water vapour and liquid water coexist in thermodyanics equilibrium in a closed container. If μ and S represent the chemical potential and entropy respectively, then at the interface.

(a) $\mu_{vapour} = \mu_{liquid}$ (b) $S_{vapour} = S_{liquid}$
(c) $S_{vapour} < S_{liquid}$ (d) $\mu_{vapour} > \mu_{liquid}$

16. Finite changes in a second order phase transition occur in
(a) S (b) V (c) C_P (d) T
Where the symbols have their standard meaning.

17. The pressure versus temperature diagram for a pure substance is shown below.



The region represented by A, B & C respectively are
(a) Solid, liquid and vapour (b) Vapour, solid and liquid
(c) Solid, vapour and liquid (d) Vapour liquid and solid.

18. The transition from ordinary liquid helium (He I) to superfluid helium (He II) is an example of a second order lambda point transition. The quantities that remain constant in such a transition at the transition point are
(a) entropy, volume, specific heat at constant pressure and Gibbs free energy
(b) pressure, volume and isothermal compressibility.
(c) specific heat at constant pressure, isothermal compressibility and pressure.
(d) pressure, volume, entropy and Gibbs free energy.

19. When two phases 1 and 2 of a system are in thermodynamic equilibrium, their temperature T, pressure p and chemical potential μ satisfy:

(a) $T_1 = T_2,\ p_1 = p_2$ and $\mu_1 = \mu_2$ (b) $T_1 = T_2,\ p_1 = p_2$ and $\mu_1 \neq \mu_2$

(c) $T_1 = T_2,\ \mu_1 = \mu_2$ and $p_1 \neq p_2$ (d) $T_1 = T_2$ but $p_1 \neq p_2,\ \mu_1 \neq \mu_2$

20. When a crystal melts into a liquid at constant pressure, which of the following quantities changes continuously at the phase transition?
(a) Gibbs free energy (b) Volume (c) Internal energy (d) Entropy

21. A solid melts into a liquid via first order phase tarnsition. The relationship between the pressure P and the temperature T of the phase transition is $P = -2T + P_0$, where P_0 is a constant. The entropy change associated with the phase transition is 1.0 J mole^{-1} K^{-1}. The Clausius-Clapeyron equation for the latent heat is $L = T\left(\frac{dP}{dT}\right)\Delta V$. Here $\Delta v = v_{liquid} - v_{solid}$ is the change in molar volume at the phase transition. The correct statement relating the values of the volumes is:

(a) $v_{liquid} = v_{solid}$ (b) $v_{liquid} = v_{solid} - 1$ (c) $v_{liquid} = v_{solid} - \frac{1}{2}$ (d) $v_{liquid} = v_{solid} + 2$

22. Experimental measurements of heat capacity per mole of aluminium at low temperatures show that the data can be fitted to the formula, $C_V = aT + bT^3$, where $a = 0.00135$ J K^{-2} mole^{-1}, $b = 2.48\times10^{-5}$ J K^{-4} mole^{-1} and T is the temperature in Kelvin. The entropy of a mole of Aluminium at such temperatures is given by the formula.

(a) $aT + \frac{b}{3}T^3 + c$, where $c > 0$ is a constant. (b) $\frac{aT}{2} + \frac{b}{4}T^3 + c$, where $c > 0$ is a constant.

(c) $aT + \frac{b}{3}T^3$ (d) $\frac{aT}{2} + \frac{b}{4}T^3$

23. A box containing 2 moles of diatomic ideal gas at temperature T_0 is connected to another identical box containing 2 moles of a monoatomic ideal gas at temperature $5T_0$. There are no thermal losses and the heat capacity of the boxes is negligible. Find the final temperature of the mixture of the gases (ignore the vibrational degrees of freedom for the diatomic molecules)

(a) T_0 (b) $1.5\,T_0$ (c) $2.5\,T_0$ (d) $3\,T_0$.

24. A gas of molecules each having mass 'm' is in thermal equilibrium at a temperature T. Let v_x, v_y, v_z to be the Cartesian components of velocity, $\vec{v}$, of a molecule. The mean value of $\left(v_x - \alpha v_y + \beta v_z\right)^2$ is:

(a) $\left(1+\alpha^2+\beta^2\right)\frac{k_BT}{m}$ (b) $\left(1-\alpha^2+\beta^2\right)\frac{k_BT}{m}$ (c) $\left(\beta^2-\alpha^2\right)\frac{k_BT}{m}$ (d) $\left(\alpha^2+\beta^2\right)\frac{k_BT}{m}$

25. A solid metallic cube of heat capacity S is at temperature 300K. It is brought in contact with a reservoir at 600K. If the heat transfer takes place only between the reservoir and the cube, the entropy change of the universe after reaching the thermal equilibrium is

(a) 0.69 S (b) 0.54 S (c) 0.27 S (d) 0.19 S

26. A real gas has specific volume v at temperature T. Its coefficient of volume expansion and isothermal compressibility are α and k_T, respectively. Its molar specific heat at constant pressure C_p and molar specific heat at constant volume C_v are related as

(a) $C_p = C_v + R$ (b) $C_p = C_v + \frac{Tv\alpha}{k_T}$ (c) $C_p = C_v + \frac{Tv\alpha^2}{k_T}$ (d) $C_p = C_v$

27. An ideal gas expands in a process in which its pressure depends on the volume as $p = p_0 e^{-\alpha V}$, where p_0 and α are two positive constants. If n is the number of moles in the gas, the maximum temperature that it will attain in this process is (R is the universal gas constant)

(a) $p_0 / e\alpha nR$ (b) $p_0 / \alpha n^3 R$ (c) $ep_0 / \alpha nR$ (d) $p_0^2 / \alpha nR$

28. An ideal gas undergoes an isothermal change of volume. The initial and final volumes are given to be 1.0 litre and 2.7 litres respectively. If in this process the entropy per mole changes from S_1 to S_2. The value of differences $S_2 - S_1$ is nearest to (R is the universal gas constant).

(a) +2.7 R (b) +1.7R (c) +1.0 R (d) –1.0R

29. A gas at a pressure P_A and volume V_A is compressed adiabatically to a volume V_B at pressure p_B. If the pressure-volume relation for this gas during adiabatic compression is $pV^{5/3} = K$ (where K is a constant), the work done during the compression is:

(a) $K\left(\frac{2}{V_B^{2/3} - V_A^{4/3}}\right)$ (b) $\frac{5}{2}K\left(\frac{2}{V_B^{2/3} - V_A^{2/3}}\right)$ (c) $\frac{3}{2}(p_B V_B - p_A V_A)$ (d) $\frac{5}{3}(p_B V_B - p_A V_A)$

30. The equation of state of an ideal gas is $p = nk_BT$, where 'p' is the thermodynamic pressure and $n = N/V$ is the thermodynamic variable for the number of particle per unit volume. The 'n' dependence of the free energy '*f*' per unit volume of the ideal gas is obtained by the following expression, where C_T is temperature-dependent constant.

(a) $nk_BT[\ln n + C_T]$ (b) $2nk_BT[n \ln n + C_T]$ (c) $\frac{3}{2}nk_BT$ (d) $3nk_BT$

31. Suppose you measured v_x (the *x*-component of the velocity) of every atom in a one-litre can of helium gas (^{4}He), kept in equilibrium at 300 K and at atmospheric pressure, at a particular instant of time. The standard deviation $\sqrt{\left\langle v_x^2 - \langle v_x \rangle^2 \right\rangle}$ in the measurement is approximately equal to

(a) 450 m/s (b) 1120 m/s (c) 1400 m/s (d) 800 m/s (e) 14000 m/s

32. A thermally 'insulated box of volume 2V has a partition which divides it into two chambers of equal volume. Each chamber contains He gas at temperature T and pressure P. On removing the partition, the molecules in the two chambers mix with each other. The change in entropy of the system is

(a) $-\frac{PV}{T}\ell n\,2$ (b) 0 (c) $\frac{PV}{T}\ell n2$ (d) $\frac{2PV}{T}\ell n2$

33. A mixture contains the same number of moles of two ideal gases A and B, with adiabatic constants γ_A and γ_B respectively. The adiabatic constant γ of the mixture satisfies

(a) $\frac{1}{\gamma-1}=\frac{1}{2}\left(\frac{1}{\gamma_A-1}+\frac{1}{\gamma_B-1}\right)$ (b) $\frac{1}{\gamma-1}=\frac{1}{2}\left(\frac{1}{\gamma_A-1}-\frac{1}{\gamma_B-1}\right)$

(c) $\frac{1}{\gamma+1}=\frac{1}{2}\left(\frac{1}{\gamma_A+1}+\frac{1}{\gamma_B+1}\right)$ (d) $\frac{1}{\gamma+1}=\frac{1}{2}\left(\frac{1}{\gamma_A+1}-\frac{1}{\gamma_B+1}\right)$

34. Two solid blocks, one at the temperature T_1 and the other T_2 ($T_1 > T_2$), with the same temperature independent heat capacity C are put in contact with each other. The change in entropy of the universe after they have equilibrated is:

(a) $C\,\ell n\left[\frac{(T_1+T_2)^2}{4T_1T_2}\right]$ (b) $C\,\ell n\left(\frac{T_1}{T_2}\right)$ (c) $C\,\ell n\left(\frac{T_1-T_2}{T_1+T_2}\right)$ (d) C

35. N_1 moles of an ideal monoatomic gas are mixed with N_2 moles of an ideal diatomic gas. If the mixture is allowed to expand adiabatically, the relation between its pressure and volume is $PV^{\gamma} = \text{constant}$, where γ is equal to

(a) $5/3$ (b) $\frac{\frac{5}{3}N_1+\frac{7}{5}N_2}{N_1+N_2}$ (c) $\frac{5N_1+7N_2}{3N_1+5N_2}$ (d) $\frac{N_1+N_2}{\frac{3}{5}N_1+\frac{5}{7}N_2}$

36. A certain gas has equation of state $p=\alpha N^2T/V^2$, where p is the pressure, N, the number of moles 'V' the volume and T, the temperature. α is a constant. One mole of the gas undergoes expansion from volume 'V' to '2V' at constant temperature T. Given that $\left(\frac{\partial S}{\partial V}\right)_{T,N}=\left(\frac{\partial p}{\partial T}\right)_{V,N,}$ the change in energy in the isothermal expansion is

(a) 0 (b) $\frac{\alpha T}{V}$ (c) $\frac{-\alpha T}{V}$ (c) $\frac{\alpha T}{V}\ell n\,2$

37. The intensity vs wavelength distribution of a blackbody is found to have its maximum at the wavelength $\lambda_{max}=0.2$ cm. Moreover, the total energy radiated J is measured to be 30 watts. If the temperature of the blackbody is now doubled, the new values of λ_{max} and J will be

(a) 0.1 cm and 120 watts (b) 0.1 cm and 240 watts.

(c) 0.1 cm and 480 watts (d) 0.4 cm an 60 watts

ANSWER KEY

1. (d)	**2. (a)**	**3. (c)**	**4. (c)**	**5. (b)**	**6. (d)**	**7. (c)**
8. (b)	**9. (b)**	**10. (b)**	**11. (c)**	**12. (b)**	**13. (a)**	**14. (a)**
15. (a)	**16. (c)**	**17. (b)**	**18. (d)**	**19. (a)**	**20. (a)**	**21. (c)**
22. (a)	**23. (c)**	**24. (a)**	**25. (d)**	**26. (c)**	**27. (a)**	**28. (c)**
29. (c)	**30. (d)**	**31. (d)**	**32. (c)**	**33. (a)**	**34. (a)**	**35. (c)**
36. (a)	**37. (c)**					

GATE Previous Years' Questions

1. A cavity of volume V contains blackbody radiation at an absolute temperature T. Which of the following option is incorrect? **[GATE 1991]**
(a) The internal energy of the radiation is proportional to T^4.
(b) The specific heat of radiation is proportional to T^3.
(c) The pressure of the radiation is proportional to T^4
(d) Although the radiation is in thermal equailibrium, the number of photons in the cavity is not constant in time.

2. Suppose temperature of the sun goes down by a factor of two, then the total power emitted by the sun will go down by a factor of **[GATE 1997]**
(a) 2 (b) 4 (c) 8 (d) 16

3. A system has N distinguishable particles. Each particle can occupy one of the two non-degenerate states with an energy difference of 0.1 eV. If the system is in thermal equilibrium at room temperature, the approximate fraction of particles in the higher energy state is **[GATE 1998]**
(a) exp (–10) (b) exp (–4) (c) exp (–2) (d) zero

4. Bose Einstein condensation temperature T_B refers to the temperature below which **[GATE 1998]**
(a) an assembly of Bose gas condenses to the liquid state.
(b) there is an appreciable occupation of the ground state in an electron system.
(c) there is a significantly large occupancy of the ground state in a system of bosons
(d) the bosons essentially behave like fermious.

5. Choose the correct alternatives **[GATE 1999]**
At the same temperature
(a) a fermion gas will exert the greatest pressure
(b) a boson gas will exert the greatest pressure
(c) a fermion gas will exert the least pressure
(d) a boson gas will exert the least pressure

6. The rotational partition function of a diatomic molecule of moment of inertia I at a temperature T is given by
(a) $\frac{Ik_BT}{\hbar^2}$ (b) $\frac{2Ik_BT}{\hbar^2}$ (c) $\frac{3Ik_BT}{\hbar^2}$ (d) $\frac{Ik_BT}{2\hbar^2}$ **[GATE 2001]**

7. Which of the following conditions should be satisfied by the temperature T of a system of N non-interacting particles occupying a volume V, for Bose-Einstein condensation to take place? **[GATE 2002]**

(a) $T < \frac{h^2}{2\pi mk_B}\left\{\frac{N}{V\varsigma\left(\frac{3}{2}\right)}\right\}^{2/3}$ (b) $T < \frac{h^2}{2\pi mk_B}\left\{\frac{N}{N\varsigma\left(\frac{3}{2}\right)}\right\}^{3/2}$

(c) $T < \frac{h^2}{2\pi mk_B}\left\{\frac{N}{V\varsigma\left(\frac{3}{2}\right)}\right\}^{1/2}$ (d) $T < \frac{h^2}{2\pi mk_B}\left\{\frac{N}{N\varsigma\left(\frac{3}{2}\right)}\right\}^{1/2}$

where m is the mass of each particle of the system, k_B is the Boltzmann constant, h is the Planck's constant and ς is the well known Zeta function.

8. Consider the Fermi-Dirac distribution function f(E) at room temperature (300 K) where E refers to energy. If E_F is the Fermi energy, which of the following is true? **[GATE 2003]**
(a) f(E) is a step function
(b) $f(E_F)$ has a value of 1/2
(c) States with $E< E_F$ are filled completely
(d) f(E) is large and tends to infinity as E decreases much below E_F.

9. Consider black body radiation in a cavity maintained at 2000 K. If the volume of the cavity is reversibly and adiabatically increased from 10 cm^3 to 640 cm^3, the temperature of the cavity changes to
(a) 800 K (b) 700 K (c) 600 K (d) 500 K **[GATE 2004]**

10. The number of states for a system of N identical free particles in a three dimensional space having total energy between E and $E+\delta E\left(\delta E << E\right)$, is proportional to **[GATE 2005]**
(a) $\left(E^{\frac{3N}{2}-1}\right)\delta E$ (b) $E^{\frac{N}{2}}\delta E$ (c) $NE^{\frac{1}{2}}\delta E$ (d) $N^N\delta E$

11. The total number of accessible states of N non intracting particles of spin 1/2 is **[GATE 2005]**
(a) 2^N (b) N^2 (c) $2N^{N/2}$ (d) N

12. The dimension of phase space of ten rigid diatomic molecules is: **[GATE 2005]**
(a) 5 (b) 10 (c) 50 (d) 100

13. A one dimensional random walker takes steps to left or right with equal probability. The probability that the random walker starting from origin is back to origin after N even number of steps is: **[GATE 2005]**
(a) $\dfrac{N!}{\left(\frac{N}{2}\right)!\left(\frac{N}{2}\right)!}\left(\frac{1}{2}\right)^N$ (b) $\dfrac{N!}{\left(\frac{N}{2}\right)!\left(\frac{N}{2}\right)!}$ (c) $2N!\left(\frac{1}{2}\right)^{2N}$ (d) $N!\left(\frac{1}{2}\right)^N$

14. If the partition function of a harmonic oscillator with frequency ω at a temperature T is $\dfrac{kT}{\hbar\omega}$, then the free energy of N such independent oscillators is: **[GATE 2005]**
(a) $\frac{3}{2}NkT$ (b) $kT\ln\frac{\hbar\omega}{kT}$ (c) $NkT\ln\frac{\hbar\omega}{kT}$ (d) $NkT\ln\frac{\hbar\omega}{2kT}$

15. The partition function of two Bose particles each of which can occupy any of the two energy levels 0 and ε is:
(a) $1+e^{-2\varepsilon/kT}+2e^{-\varepsilon/kT}$ (b) $1+e^{-2\varepsilon/kT}+e^{-\varepsilon/kT}$ (c) $2+e^{-2\varepsilon/kT}+e^{-\varepsilon/kT}$ (d) $e^{-2\varepsilon/kT}+e^{-\varepsilon/kT}$ **[GATE 2005]**

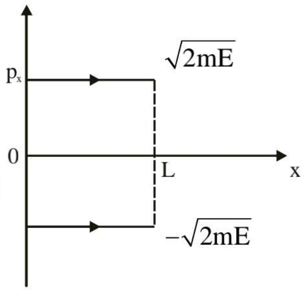
16. A system of non-interacting Fermi particles with Fermi energy E_F has the density of states proportional to $\sqrt{E}$, where E is the energy of a particle. The average energy per particle at temperature $T=0$ is:
(a) $\frac{1}{6}E_F$ (b) $\frac{1}{5}E_F$ (c) $\frac{2}{5}E_F$ (d) $\frac{3}{5}E_F$ **[GATE 2005]**

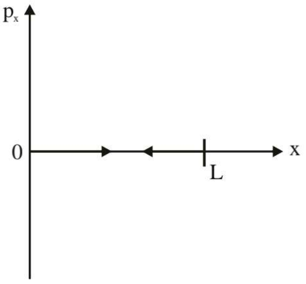
17. The pressure for a non-interacting Fermi gas with internal energy U at temperature T is: **[GATE 2005]**
(a) $P=\frac{3}{2}\frac{U}{V}$ (b) $P=\frac{2}{3}\frac{U}{V}$ (c) $P=\frac{3}{5}\frac{U}{V}$ (d) $P=\frac{1}{2}\frac{U}{V}$

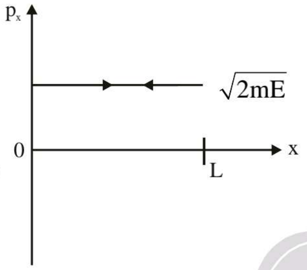
18. A monatomic crystalline solid comprises of N atoms, out of which n atoms are in interstitial positions. If the available interstitial sites are N', then number of possible microstates is: **[GATE 2006]**
(a) $\dfrac{(N'+n)!}{n!N!}$ (b) $\dfrac{(N+n)!}{n!(N'-n)!}\dfrac{(N'+n)!}{n!(N'-n)!}$ (c) $\dfrac{N!}{n!(N'-n)!}$ (d) $\dfrac{N!}{n!(N-n)!}\dfrac{N'!}{n!(N'-n)!}$

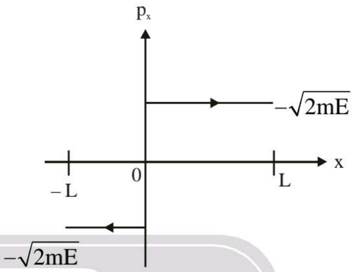
19. Each of the two isolated vessels, A and B of fixed volumes, contains N molecules of a perfect monatomic gas at a pressure P. The temperature of A and B are T_1 and T_2, respectively. The two vessels are brought into thermal contct. At equlibrium, the change in entropy is: **[GATE 2006]**
(a) $\frac{3}{2}Nk_B\ln\left[\frac{T_1^2+T_2^2}{4T_1T_2}\right]$ (b) $Nk_B\ln\left(\frac{T_2}{T_1}\right)$
(c) $\frac{3}{2}Nk_B\ln\left[\frac{(T_1+T_2)^2}{4T_1T_2}\right]$ (d) $2Nk_B$

20. The phase diagram of a free particle of mass m and kinetic energy E, moving in one-dimensional box with perfectly elastic walls at x = 0 and x = L, is given by **[GATE 2006]**

(a)


(b)


(c)


(d)


21. A system of N localized, non-interacting spin $\frac{1}{2}$ ions of magnetic moment μ each is kept in an external magnetic field H. If the system is in equilibrium at temperature T, then Helmholtz free energy of the system is: **[GATE 2006]**

(a) $Nk_BT\ln\left(\cos h\frac{\mu H}{k_BT}\right)$ (b) $-Nk_BT\ln\left(2\cos h\frac{\mu H}{k_BT}\right)$

(c) $Nk_BT\ln\left(2\cos h\frac{\mu H}{k_BT}\right)$ (d) $-Nk_BT\ln\left(2\sin h\frac{\mu H}{k_BT}\right)$

22. The free energy of a photon gas enclosed in a volume V is given by $F=-\frac{1}{3}aVT^4$ where *a* is a constant and T is the temperature of the gas. The chemical potential of the photon gas is **[GATE 2006]**

(a) 0 (b) $\frac{4}{3}aVT^3$ (c) $\frac{1}{3}aVT^3$ (d) aVT^{-4}

23. A system has energy level $E_0, 2E_0, 3E_0, ...$, where the excited states are triply degenerate. Four non-interacting bosons are placed in this system. If the total energy of these bosons is $5E_0$, the number of microstates is: **[GATE 2007]**

(a) 2 (b) 3 (c) 4 (d) 5

24. It is necessary to apply quantum statistics to a system of particles if **[GATE 2007]**
(a) There is substaintial overlap between the wavefunctions of the particles.
(b) The mean free path of the particles is comparable to the inter-particle separation.
(c) The particles have identical mass and charge.
(d) The particles are interacting.

25. The free energy for a photon gas is given by $F = -\left(\frac{a}{3}\right)VT^4$, where *a* is a constant. The entropy S and the pressure P of the photon gas are **[GATE 2007]**

(a) $S = \frac{4}{3}aVT^3, P = \frac{a}{3}T^4$
(b) $S = \frac{1}{3}aVT^4, P = \frac{4a}{3}T^3$
(c) $S = \frac{4}{3}aVT^4, P = \frac{a}{3}T^3$
(d) $S = \frac{1}{3}aVT^3, P = \frac{4a}{3}T^4$

26. Thermodynamic variables of a system can be volume V, pressure P, temperature T, number of particles N, internal energy E and chemical potential μ, etc. For a system to be specified by microcanonical (MC), Canonical (CE) and Grand Canonial (GC) ensembles, the parameters required for the respective ensembles are

(a) $MC:(N,V,T); CE(E,V,N); GC:(V,T,\mu)$ **[GATE 2008]**
(b) $MC:(E,V,N); CE:(N,V,T); GC:(V,T,\mu)$
(c) $MC:(V,T,\mu); CE:(N,V,T); GC:(E,V,N)$
(d) $MC:(E,V,N); CE:(V,T,\mu); GC:(N,V,T)$

27. A photon gas is at thermal equilibrium at temperature T. The mean number of photons in an energy state $\varepsilon = \hbar\omega$ is: **[GATE 2008]**

(a) $\exp\left(\frac{\hbar\omega}{k_BT}\right)+1$ (b) $\exp\left(\frac{\hbar\omega}{k_BT}\right)-1$ (c) $\left(\exp\left(\frac{\hbar\omega}{k_BT}\right)+1\right)^{-1}$ (d) $\left(\exp\left(\frac{\hbar\omega}{k_BT}\right)-1\right)^{-1}$

28. Two identical particles have to be distributed among three energy levels. let r_B, r_F and r_C represent the ratios of probability of finding two particles to that of finding one particle in a given enegy state. The subscripts B, E and C correspond to whether the particles are bosons, fermions and classical particles, respectively. The r_B:r_F:r_C is equal to **[GATE 2008]**

(a) $\frac{1}{2}:0:1$ (b) $1:\frac{1}{2}:1$ (c) $1:\frac{1}{2}:\frac{1}{2}$ (d) $1:0:\frac{1}{2}$

29. Consider a system of two non-interacting classical particles which can occupy any of the three energy levels with energy values $E = 0$, ε and 2ε having degeneracies $g(E) = 1, 2$ and 4 respectively. The mean energy of the system is: **[GATE 2008]**

(a) $\varepsilon\left(\frac{4\exp\left(-\varepsilon/k_BT\right)+8\exp\left(-2\varepsilon/k_BT\right)}{1+2\exp\left(-\varepsilon/k_BT\right)+4\exp\left(-2\varepsilon/k_BT\right)}\right)$
(b) $\varepsilon\left(\frac{2\exp\left(-\varepsilon/k_BT\right)+8\exp\left(-2\varepsilon/k_BT\right)}{1+2\exp\left(-\varepsilon/k_BT\right)+4\exp\left(-2\varepsilon/k_BT\right)}\right)$

(c) $\varepsilon\left(\frac{2\exp\left(-\varepsilon/k_BT\right)+4\exp\left(-2\varepsilon/k_BT\right)}{1+2\exp\left(-\varepsilon/k_BT\right)+4\exp\left(-2\varepsilon/k_BT\right)}\right)^2$
(d) $\varepsilon\left(\frac{\exp\left(-\varepsilon/k_BT\right)+2\exp\left(-2\varepsilon/k_BT\right)}{1+\exp\left(-\varepsilon/k_BT\right)+\exp\left(-2\varepsilon/k_BT\right)}\right)$

30. The probability that an energy level ε at a temperature T is unoccupied by a fermion of chemical potential μ is given by

(a) $\frac{1}{e^{(\varepsilon-\mu)/k_BT}+1}$ (b) $\frac{1}{e^{(\varepsilon-\mu)/k_BT}-1}$ (c) $\frac{1}{e^{(\mu-\varepsilon)/k_BT}+1}$ (d) $\frac{1}{e^{(\mu-\varepsilon)/k_BT}-1}$

31. Consider the electron gas in a simple metal at room temperature
(a) This is a degenerate Fermi-Dirac gas.
(b) The gas displays paramagnetism owing to the spins of the electrons.
(c) All the electrons are in the same single-particle energy level.
(d) The electrons have wave functions localized about the lattice positions of the positive ions.

32. Which of the following atoms cannot exhibit Bose-Einstein condensation, even in principle? **[GATE 2010]**

(a) ${}_{1}^{1}\text{H}$ (b) ${}_{2}^{4}\text{He}$ (c) ${}_{11}^{23}\text{Na}$ (d) ${}_{19}^{40}\text{K}$

33. For a two-dimensional free electron gas, the electronic density n, and the Fermi energy E_F, are related by **[GATE 2010]**

(a) $n = \frac{(2mE_F)^{3/2}}{3\pi^2\hbar^3}$ (b) $n = \frac{mE_F}{\pi\hbar^2}$ (c) $n = \frac{mE_F}{2\pi\hbar^2}$ (d) $n = \frac{2^{3/2}(mE_F)^{1/2}}{\pi\hbar}$

34. A system of N non-interacting and distinguishable particles of spin 1 is in thermodynamic equilibrium. The entropy of the system is: **[GATE 2011]**

(a) $2k_B \ln N$ (b) $3k_B \ln N$ (c) $Nk_B \ln 2$ (d) $Nk_B \ln 3$

35. The total energy, E of an ideal non-relativistic Fermi gas in three dimensions is given by $E \propto \frac{N^{5/3}}{V^{2/3}}$ where N is the number of particles and V is the volume of the gas. Identify the CORRECT equation of state (P being the pressure), **[GATE 2012]**

(a) $PV = \frac{1}{3}E$ (b) $PV = \frac{2}{3}E$ (c) $PV = E$ (d) $PV = \frac{5}{3}E$

36. Across a first order phase transition, the free energy is **[GATE 2013]**
(a) proportional to the temperature
(b) a discontinuous function of the temperature
(c) a continuous function of the temperature but its derivative is discontinuous
(d) such that the first derivative with respect to temperature is continuous

37. Two gases separated by an impermeable but movable partition are allowed to freely exchange energy. At equilibrium, the two sides will have the same **[GATE 2013]**
(a) pressure and temperature (b) volume and temperature
(c) pressure and volume (d) volume and energy

38. The entropy function of a system is given by $S(E) = aE(E_0 - E)$ where a and E_0 are positive constants. The temperature of the system is **[GATE 2013]**
(a) negative for some energies (b) increases monotonically with energy
(c) decreases monotonically with energy (d) zero

39. Consider a linear collection of N independent spin 1/2 particles, each at a fixed location. The entropy of this system is (k is the Boltzmann constant) **[GATE 2013]**

(a) Zero (b) Nk (c) $\frac{1}{2}Nk$ (d) $Nk\, ln(2)$

40. Consider a gas of atoms obeying Maxwell-Boltzman statistics. The average value of $e^{i\vec{a}\cdot\vec{p}}$ over all the momenta $\vec{p}$ of each of the particles (where $\vec{a}$ is a constant vector and a is its magnitude, m is the mass of each atom, T is temperature and k is Boltzmann's constant) is, **[GATE 2013]**

(a) One (b) Zero (c) $e^{-\frac{1}{2}a^2mkT}$ (d) $e^{-\frac{3}{2}a^2mkT}$

41. The number of distinct ways of placing four indistinguishable balls into five distinguishable boxes is ______

42. For a gas under isothermal conditions, its pressure P varies with volume V as $P \propto V^{-5/3}$. The bulk modulus B is proportional to **[GATE 2014]**

(a) $V^{-1/2}$ (b) $V^{-2/3}$ (c) $V^{-3/5}$ (d) $V^{-5/3}$

43. At a given temperature, T, the average energy per particle of a non-interacting gas of two-dimensional classical harmoic oscillator is _____________ $k_B T$. **[GATE 2014]**

44. Consider a system of 3 fermions, which can occupy any of the 4 available energy states with equal probability. The entropy of the system is **[GATE 2014]**

(a) $k_B \ln 2$ (b) $2k_B \ln 2$ (c) $3k_B \ln 2$ (d) $3k_B \ln 4$

45. The energy dependence of the density of states for a two dimensional non-relativistic electron gas is given by, $g(E) = CE^n$, where C is constant. The value of n is _______________ **[GATE 2015]**

46. For a black body radiation in a cavity, photons are created and annihilated freely as a result of emission and absorption by the walls of the cavity. This is because **[GATE 2015]**
(a) the chemical potential of the photons is zero
(b) photons obey Pauli exclusion principle
(c) photons are spin-1 particles
(d) the entropy of the photons is very large

47. In Bose-Einstein condensation, the particles **[GATE 2015]**
(a) have strong interparticle attraction (b) condense in real space
(c) have overlapping wavefunctions (d) have large and positive chemical potential

48. For a system at constant temperature and volume, which of the following statements is correct at equilibrium?
(a) The Helmholtz free energy attains a local minimum **[GATE 2016]**
(b) The Helmholtz free energy attains a local maximum
(c) The Gibbs free energy attains a local minimum
(d) The Gibbs free energy attains a local maximum

49. The total power emitted by a spherical black body of radius R at a temperature T is P_1. Let P_2 be the total power emitted by another spherical black body of radius $R/2$ kept at temperature $2T$. The ratio P_1/P_2 is ____________. (Give your answer upto two decimal places) **[GATE 2016]**

50. A two-level system has energies zero and E. The level with zero energy is non-degenerate, while the level with energy E is triply degenerate. The mean energy of a classical particle in this system at a temperature T is **[GATE 2016]**

(a) $\dfrac{Ee^{-E/k_BT}}{1+3e^{-E/k_BT}}$ (b) $\dfrac{Ee^{-E/k_BT}}{1+e^{-E/k_BT}}$ (c) $\dfrac{3Ee^{-E/k_BT}}{1+e^{-E/k_BT}}$ (d) $\dfrac{3Ee^{-E/k_BT}}{1+3e^{-E/k_BT}}$

51. The energy density and pressure of a photon gas are given by $u = aT^4$ and $P = u/3$, where T is the temperature and a is the radiation constant. The entropy per unit volume is given by $\alpha a T^3$. The value of α is _____. (up to two decimal places). **[GATE 2017]**

52. Consider N non-interacting, distinguishable particles in a two-level system at temperature T. The energies of the levels are 0 and ε, where $\varepsilon > 0$. In the high temperature limit $(k_BT >> \varepsilon)$, what is the population of particles in the level with energy ε? **[GATE 2017]**

(a) $\dfrac{N}{2}$ (b) N (c) $\dfrac{N}{4}$ (d) $\dfrac{3N}{4}$

53. Consider two particles and two non-degenerate quantum levels 1 and 2. Level 1 always contains a particle. Hence, what is the probability that level 2 also contains a particle for each of the two cases:

(i) when the two particles are distinguishable and (ii) when the two particles are bosons?**[GATE 2017]**
(a) (i) 1/2 and (ii) 1/3 (b) (i) 1/2 and (ii) 1/2 (c) (i) 2/3 and (ii) 1/2 (d) (i) 1 and (ii) 0

54. The phase space trajectory of an otherwise free particle bouncing between two hard walls elastically in one dimension is a **[GATE 2017]**
(a) straight line (b) parabola (c) rectangle (d) circle

55. Consider a 2-dimensional electron gas with a density of $10^{19}\ \text{m}^{-2}$. The Fermi energy of the system is _____eV. (up to two decimal places). **[GATE 2017]**

$\left(m_e = 9.31\times10^{-31}\,\text{kg},\ h = 6.626\times10^{-34}\,\text{Js},\ e = 1.602\times10^{-19}\,\text{C}\right)$

56. Two solid spheres A and B have same emissivity. The radius of A is four times the radius of B, and temperature of A is twice the temperature of B. The ratio of the rate of heat radiated from A to that from B is ________ . **[GATE 2018]**

57. The energy-wavevector (E-k) dispersion relation for a particle in two dimensions is $E = Ck$, where C is a constant. If its density of states $D(E)$ is proportional to E^p then the value of p is __________. **[GATE 2019]**

ANSWER KEY

1. (d)	2. (d)	3. (d)	4. (c)	5. (a)	6. (b)	7. (a)
8. (b)	9. (d)	10. (a)	11. (a)	12. (d)	13. (a)	14. (c)
15. (b)	16. (d)	17. (b)	18. (d)	19. (c)	20. (a)	21. (b)
22. (a)	23. (b)	24. (b)	25. (a)	26. (b)	27. (d)	28. (d)
29. (b)	30. (c)	31. (a)	32. (d)	33. (b)	34. (d)	35. (b)
36. (c)	37. (a)	38. (a)	39. (d)	40. (c)	41. (70)	42. (d)
43. (1.99 to 2.01)		44. (b)	45. (0)	46. (a)	47. (a)	48. (a)
49. (0.25)	50. (d)	51. (1.30 to 1.36)		52. (a)	53. (c)	54. (c)
55. (2.32 to 2.38)		56. (256 to 256)		57. (1)		

CSIR-UGC-NET Previous Years' Questions

1. A cavity contains blackbody radiation in equilibrium at temperature T. The specific heat per unit volume of the photon gas in the cavity is of the form $C_V = \gamma T^3$ where γ is a constant. The cavity is expanded to twice its original volume and then allowed to equilibrate at the same temperature T. The new internal energy per unit volume is: **[NET June 2011]**

   (a) $4\gamma T^4$ (b) $2\gamma T^4$ (c) $\dfrac{\gamma T^4}{4}$ (d) γT^4

2. Consider an ideal Bose gas in three dimensions with the energy-momentum relation $\varepsilon \propto p^s$ with $s > 0$. The range of s for which this system may undergo a Bose-Einstein condensation at a non-zero temperature is:

   (a) $1 < s < 3$ (b) $0 < s < 2$ (c) $0 < s < 3$ (d) $0 < s < \infty$ **[CSIR June 2011]**

3. The excitation of a three-dimensional solid are bosonic in nature with their frequency ω and wave-number k are related by $\omega \propto k^2$ in the large wavelength limit. If the chemical potential is zero, the behaviour of the specific heat of the system at low temperature is proportional to **[CSIR June 2011]**

   (a) $T^{1/2}$ (b) T (c) $T^{3/2}$ (d) T^3

4. The internal energy E of a system is given by $E = \dfrac{bS^3}{VN}$, where b is a constant and other symbols have their usual meaning . The temperature of this system is equal to **[CSIR Dec. 2011]**

   (a) $\dfrac{bS^2}{VN}$ (b) $\dfrac{3bS^2}{VN}$ (c) $\dfrac{bS^3}{V^3N}$ (d) $\left(\dfrac{S}{N}\right)^2$

5. If the number density of a free electron gas in three dimensions is increased eight times, its Fermi temprature will **[CSIR Dec. 2011]**

   (a) increase by a factor of 4 (b) decrease by a factor of 4

   (c) increase by a factor of 8 (d) decrease by a factor of 8

6. Consider a system of non-interacting particles in d dimensions obeying the dispersion relation $\varepsilon = Ak^s$, where ε is the energy, *k* is the wavevector, '*s*' is an integer and *A* a constant. The density of states, $N(\varepsilon)$, is proportional to **[NET June 2012]**

   (a) $\varepsilon^{\frac{1}{d}-1}$ (b) $\varepsilon^{\frac{d}{s}+1}$ (c) $\varepsilon^{\frac{d}{s}-1}$ (d) $\varepsilon^{\frac{s}{d}-s}$

7. Consider black body radiation contained in a cavity whose walls are at temperature *T*. The radiation is in equilibrium with the walls of the cavity. If the temperature of the walls is increased to 2*T* and the radiation is allowed to come to equilibrium at the new temperature, the entropy of the radiation increases by a factor of

   (a) 2 (b) 4 (c) 8 (d) 16 **[NET June 2012]**

8. The number of ways in which N identical bosons can be distributed in two energy levels, is

   (a) $N + 1$ (b) $N(N-1)/2$ (c) $N(N+1)/2$ (d) N **[NET June 2012]**

9. The free energy of a gas *N* particled in a volume *V* and at a temperature *T* is $F = NK_BT\ln\left[a_0V\left(k_BT\right)^{5/2}/N\right]$, where a_0 is a constant and k_B denotes the Boltzmann constant. The internal energy of gas is

   (a) $\dfrac{3}{2}Nk_BT$ (b) $\dfrac{5}{2}Nk_BT$ **[NET June 2012]**

   (c) $Nk_BT\ln\left[a_0V\left(k_BT\right)^{5/2}/N\right]-\dfrac{3}{2}Nk_BT$ (d) $Nk_BT\ln\left[a_0V/\left(k_BT\right)^{5/2}\right]$

10. The entropy of a system, S, is related to the accessible phase space volume Γ by $S = k_B \ \ell n \, \Gamma(E, N, V)$ where E, N and V are the energy, number of particles and volume respectively. From this one can conclude that Γ **[CSIR Dec. 2012]**

(a) does not change during evolution to equilibrium
(b) Oscillates during evolution to equilibrium
(c) Is a maximum in equilibrium
(d) Is a minimum in equilibrium

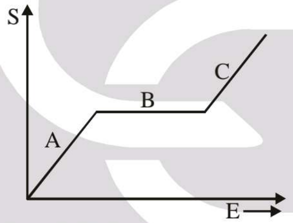
11. If the energy dispersion of a two-dimensional electron system is $E = u\hbar k$ where u is the velocity and k is the momentum, then the density of states D(E) depends on the energy as **[CSIR June 2013]**

(a) $1/\sqrt{E}$ (b) $\sqrt{E}$ (c) E (d) constant

12. Two different thermodynamic systems are described by the following equations of state: $\frac{1}{T^{(1)}} = \frac{3RN^{(1)}}{2U^{(1)}}$ and $\frac{1}{T^{(2)}} = \frac{5RN^{(2)}}{2U^{(2)}}$ where $T^{(1,2)}, N^{(1,2)}$ and $U^{(1,2)}$ are respectively, the temperatures, the mole numbers and the internal energies of the two systems, and R is the gas constant. Let U_{tot} denote the total energy when these two systems are put in contact and attain thermal equilibrium. The ratio $\frac{U^{(1)}}{U_{tot}}$ is **[NET Dec. 2013]**

(a) $\frac{5N^{(2)}}{3N^{(1)}+5N^{(2)}}$ (b) $\frac{3N^{(1)}}{3N^{(1)}+5N^{(2)}}$ (c) $\frac{N^{(1)}}{N^{(1)}+N^{(2)}}$ (d) $\frac{N^{(2)}}{N^{(1)}+N^{(2)}}$

13. The entropy S of a thermodynamic system as a function of energy E is given by the following graph



The temperatures of the phases A, B and C, denoted by T_A, T_B and T_C, respectively, satisfy the following inequalities: **[NET Dec. 2013]**

(a) $T_C > T_B > T_A$ (b) $T_A > T_C > T_B$ (c) $T_B > T_C > T_A$ (d) $T_B > T_A > T_C$

14. Consider the melting transition of ice into water at constant pressure. Which of the following thermodynamic quantities does not exhibit a discontinuous change across the phase transition? **[CSIR Dec. 2013]**

(a) internal energy (b) Helmholtz free energy
(c) Gibbs free energy (d) entropy

15. The pressure of a non-relativistic free Fermi gas in three-dimensions depends, at T = 0, on the density of fermions n as **[CSIR June 2014]**

(a) $n^{5/3}$ (b) $n^{1/3}$ (c) $n^{2/3}$ (d) $n^{4/3}$

16. A system of N distinguishable particles, each of which can be in one of the two energy levels 0 and e, has a total energy ne, where n is an integer. The entropy of the system is proportional to **[NET June 2015]**

(a) $N \ell n \, n$ (b) $n \ell n \, N$ (c) $\ell n\left(\frac{N!}{n!}\right)$ (d) $\ell n\left(\frac{N!}{n!(N-n)!}\right)$

17. For a system of independent non-interacting one-dimensional oscillators, the value of the free energy per oscillator, in the limit $T \to 0$, is **[NET Dec. 2015]**

(a) $\frac{1}{2}\hbar\omega$ (b) $\hbar\omega$ (c) $\frac{3}{2}\hbar\omega$ (d) 0

18. Consider a random walker on a square lattice. At each step the walker moves to a nearest neighbour site with equal probability for each of the four sites. The walker starts at the origin and takes 3 steps. The probability that during this walk no site is visited more than once is **[NET Dec. 2015]**

(a) 12/27 (b) 27/64 (c) 3/8 (d) 9/16

19. The internal energy $E(T)$ of a system at a fixed volume is found to depend on the temperature T as $E(T) = aT^2 + bT^4$. Then the entropy $S(T)$, as a function of temperature, is **[NET June 2016]**

(a) $\frac{1}{2}aT^2 + \frac{1}{4}bT^4$ (b) $2aT^2 + 4bT^4$ (c) $2aT + \frac{4}{3}bT^3$ (d) $2aT + 2bT^3$

20. A thermodynamic function $G(T, P, N) = U - TS + PV$ is given in terms of the internal energy U, temperature T, entropy S, pressure P, volume V and the number of particles N. Which of the following relations is true ? (In the following μ is the chemical potential). **[NET June 2017]**

(a) $S = -\left.\frac{\partial G}{\partial T}\right|_{N,P}$ (b) $S = \left.\frac{\partial G}{\partial T}\right|_{N,P}$ (c) $V = -\left.\frac{\partial G}{\partial P}\right|_{N,T}$ (d) $\mu = -\left.\frac{\partial G}{\partial N}\right|_{P,T}$

21. A gas of photons inside a cavity of volume V is in equilibrium at temperature T. If the temperature of the cavity is changed to $2T$, the radiation pressure will change by a factor of **[NET June 2017]**

(a) 2 (b) 16 (c) 8 (d) 4

22. In a thermodynamic system in equilibrium, each molecule can exist in three possible states with probabilities $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{6}$ respectively. The entropy per molecule is **[NET June 2017]**

(a) $k_B \ln 3$ (b) $\frac{1}{2}k_B \ln 2 + \frac{2}{3}k_B \ln 3$

(c) $\frac{2}{3}k_B \ln 2 + \frac{1}{2}k_B \ln 3$ (d) $\frac{1}{2}k_B \ln 2 + \frac{1}{6}k_B \ln 3$

23. Consider a quantum system of non-interacting bosons in contact with a particle bath. The probability of finding no particle in a given single particle quantum state is 10^{-6}. The average number of particles in that state is of the order of

(a) 10^3 (b) 10^6 (c) 10^9 (d) 10^{12} **[NET Dec. 2017]**

24. A closed system having three non-degenerate energy levels with energies $E = 0, \pm\varepsilon$, is at temperature T. For $\varepsilon = 2k_BT$, the probability of finding the system in the state with energy $E = 0$, is **[NET Dec. 2017]**

(a) $\frac{1}{(1+2\cosh 2)}$ (b) $\frac{1}{(2\cosh 2)}$ (c) $\frac{1}{2}\cosh 2$ (d) $\frac{1}{\cosh 2}$

25. The relation between the internal energy U, entropy S, temperature T, pressure p, volume V, chemical potential μ and number of particles N of a thermodynamic system is $dU = TdS - pdV + \mu dN$. That U is an exact differential implies that **[NET Dec. 2017]**

(a) $-\left.\frac{\partial p}{\partial S}\right|_{V,N} = \left.\frac{\partial T}{\partial V}\right|_{S,N}$ (b) $p\left.\frac{\partial U}{\partial T}\right|_{S,N} = S\left.\frac{\partial U}{\partial V}\right|_{S,\mu}$

(c) $p\left.\frac{\partial U}{\partial T}\right|_{S,N} = -\frac{1}{T}\left.\frac{\partial U}{\partial V}\right|_{S,\mu}$ (d) $\left.\frac{\partial p}{\partial S}\right|_{V,N} = \left.\frac{\partial T}{\partial V}\right|_{S,N}$

26. Which of the following statements concerning the coefficient of volume expansion α and the isothermal compressibility κ of a solid is true ? **[NET June 2018]**

(a) α and κ are both intensive variables.

(b) α is an intensive and κ is an extensive variable.

(c) α is an extensive and κ is an intensive variable.

(d) α and κ are both extensive variables.

27. The number of ways of distributing 11 indistinguishable bosons in 3 different energy levels is

(a) 3^{11} (b) 11^3 (c) $\frac{(13)!}{2!(11)!}$ (d) $\frac{(11)!}{3!8!}$ **[NET June 2018]**

28. In a system of N distinguishable particles, each particle can be in one of two states with energies 0 and $-E$, respectively. The mean energy of the system at temperature T, is **[NET June 2018]**

(a) $-\frac{1}{2}N\left(1+e^{E/k_BT}\right)$ (b) $-\frac{NE}{\left(1+e^{E/k_BT}\right)}$ (c) $-\frac{1}{2}NE$ (d) $-\frac{NE}{\left(1+e^{-E/k_BT}\right)}$

29. The heat capacity C_V at constant volume of a metal, as a function of temperature, is $\alpha T+\beta T^3$, where α and β are constants. The temperature dependence of the entropy at constant volume is **[NET Dec. 2018]**

(a) $\alpha T+\frac{1}{3}\beta T^3$ (b) $\alpha T+\beta T^3$ (c) $\frac{1}{2}\alpha T+\frac{1}{3}\beta T^3$ (d) $\frac{1}{2}\alpha T+\frac{1}{4}\beta T^3$

30. In a system comprising of approximately 10^{23} distinguishable particles, each particle may occupy any of 20 distinct states. The maximum value of the entropy per particle is nearest to **[NET June 2019]**

(a) $20k_B$ (b) $3k_B$ (c) $10(\ln 2)k_B$ (d) $20(\ln 2)k_B$

31. Consider black body radiation in thermal equilibrium contained in a two-dimensional box. The dependence of the energy density on the temperature T is **[NET Dec. 2019]**

(a) T^3 (b) T (c) T^2 (d) T^4

32. For T much less than the Debye temperature of copper, the temperature dependence of the specific heat at constant volume of copper, is given by (in the following a and b are positive constants) **[NET Dec. 2019]**

(a) aT^3 (b) $aT+bT^3$ (c) aT^2+bT^3 (d) $\exp\left(-\frac{a}{k_BT}\right)$

33. There are n distinguishable balls which are to be put in n boxes, where each box can contain at most $(n-1)$ balls. The number of ways in which the balls can be distributed in the boxes is **[NET Dec. 2019]**

(a) $n(n^{n-1}-1)$ (b) $(n-1)^n$ (c) n^{n-1} (d) $n!-n$

ANSWER KEY

1. (c)	2. (c)	3. (c)	4. (b)	5. (a)	6. (c)	7. (c)
8. (a)	9. (b)	10. (c)	11. (c)	12. (b)	13. (c)	14. (c)
15. (a)	16. (d)	17. (a)	18. (d)	19. (c)	20. (a)	21. (b)
22. (c)	23. (b)	24. (a)	25. (a)	26. (a)	27. (c)	28. (d)
29. (a)	30. (b)	31. (a)	32. (b)	33. (a)		

JEST Previous Years' Questions

1. A collection of N two- level systems with energies 0 and $E > 0$ is in thermal equilibrium at temperature T. For $T \to \infty$, the specific. heat approaches **[JEST 2012]**

(a) 0 (b) Nk_B (c) $3Nk_{B/2}$ (d) ∞

2. Consider a particle with three possible spin statesL $s = 0$and ± 1. There is a magnetic field h present and the energy for a spin state s is –hs. The system is at a temperature T. Which of the following statements is true about the entropy S(T)? **[JEST 2013]**

(a) S(T) = In 3 at T = 0, and 3 at high T (b) S(T) = In 3 at T 0, and zero at high T
(c) S(T) = 0 at T = 0, and 3 at high T (d) S(T) = 0 for T = 0, and In 3 at high T

3. Consider three situations of 4 particles in a one dimensional box of width L with hard walls. In case (i), the particles are fermions, in case (ii) they are bosons, and in case (iii) they are classical. If the total ground state energy of the four particles in these three cases are E_F, E_B and E_{cl} respectively, which of the following is true?

(a) $E_F = E_B = E_{Cl}$ (b) $E_F > E_B = E_{Cl}$ (c) $E_F < E_B < E_{Cl}$ (d) $E_F > E_B > E_{Cl}$ **[JEST 2013]**

4. An ideal gas of non-relativistic fermions in 3-dimensions is at 0K. When both the number density and mass of the partiles are doubled, then the energy per particle is multiplied by a factor **[JEST 2014]**

(a) $2^{1/2}$ (b) 1 (c) $2^{1/3}$ (d) $2^{-1/3}$

5. Consider a system of 2N non-interacting spin 1/2 particles each fixed in position and carrying a magnetic moment μ. The system is immersed in a uniform magnetic field B. The number of spin up particle for which the entropy of the system will be maximum is **[JEST 2014]**

(a) 0 (b) N (c) 2N (d) N/2

6. For a system in thermal equilibrium with a heat bath at temperature T, which one of the following equalities is correct ? $\left(\beta = \frac{1}{k_B T}\right)$ **[JEST 2015]**

(a) $\frac{\partial}{\partial\beta}\langle E\rangle = \langle E\rangle^2 - \langle E^2\rangle$ (b) $\frac{\partial}{\partial\beta}\langle E\rangle = \langle E^2\rangle - \langle E\rangle^2$

(c) $\frac{\partial}{\partial\beta}\langle E\rangle = \langle E^2\rangle + \langle E\rangle^2$ (d) $\frac{\partial}{\partial\beta}\langle E\rangle = -\left(\langle E^2\rangle + \langle E\rangle^2\right)$

7. The blackbody at a temperature of 6000 K emits a radiation whose intensity spectrum peaks at 600 nm. If the temperature is reduced to 300 K, the spectrum will peak at, **[JEST 2015]**

(a) 120 μm (b) 12 μm (c) 12 mm (d) 120 mm

8. For a quantum mechanical harmonic oscillator with energies, $E_n = (n + 1/2)\hbar\omega$, where $n = 0, 1, 2, ...,$ the partition function is: **[JEST 2016]**

(a) $\frac{e^{\hbar\omega/k_BT}}{e^{\hbar\omega/k_BT} - 1}$ (b) $e^{\hbar\omega/2k_BT} - 1$ (c) $e^{\hbar\omega/2k_BT} + 1$ (d) $\frac{e^{\hbar\omega/2k_BT}}{e^{\hbar\omega/k_BT} - 1}$

9. A gas contains particles of type A with fraction 0.8, and particles of type B with fraction 0.2. The probability that among 3 randomly chosen particles at least one is of type A is: **[JEST 2016]**

(a) 0.8 (b) 0.25 (c) 0.33 (d) 0.992

10. If the mean square fluctuations in energy of a system in equilibrium at temperature T is proportional to T^{α}, then the energy of the system is proportional to **[JEST 2017]**

(a) $T^{\alpha-2}$ (b) $T^{\alpha/2}$ (c) $T^{\alpha-1}$ (d) T^{α}

11. Consider a system of N distinguishable particles with two energy levels for each particle; a ground state with energy zero and an excited state with energy $\varepsilon > 0$. What is the average energy per particle as the system temperature $T \to \infty$? **[JEST 2019]**

(a) 0 (b) $\varepsilon/2$ (c) ε (d) ∞

12. Consider a diatomic molecule with an infinite number of equally spaced non-degenerate energy levels. The spacing between any two adjacent levels is ε and the ground state energy is zero. What is the single particle partition function Z? **[JEST 2019]**

(a) $Z=\dfrac{1}{1-\dfrac{\varepsilon}{k_BT}}$ (b) $Z=\dfrac{1}{1-e^{\varepsilon/k_BT}}$ (c) $Z=\dfrac{1}{1-e^{2\varepsilon/k_BT}}$ (d) $Z=\dfrac{1-\dfrac{\varepsilon}{k_BT}}{1+\dfrac{\varepsilon}{k_BT}}$

13. The energy spectrum of a particle consists of four states with energies $0, \varepsilon, 2\varepsilon, 3\varepsilon$. Let $Z_B(T)$, $Z_F(T)$ and $Z_C(T)$ denote the canonical partition functions for four non-interacting particles at temperature T. The subscripts B, F and C corresponds to bosons, fermions and distinguishable classical particles, respectively.

Let $y \equiv \exp\left(-\dfrac{\varepsilon}{k_BT}\right)$

Which one of the following statements is true about $Z_B(T)$, $Z_F(T)$ and $Z_C(T)$? **[JEST 2019]**

(a) They are polynomials in y of degree 12, 6 and 12, respectively.
(b) They are polynomials in y of degree 16, 10 and 16, respectively.
(c) They are polynomials in y of degree 9, 6 and 12, respectively.
(d) They are polynomials in y of degree 12, 10 and 16, respectively.

14. Consider a non-relativistic two-dimensional gas of N electrons with the Fermi energy E_F. What is the average energy per particle at temperature $T=0$? **[JEST 2019]**

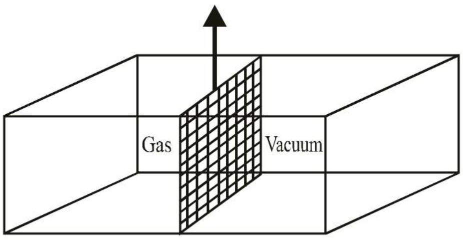
(a) $\dfrac{3}{5}E_F$ (b) $\dfrac{2}{5}E_F$ (c) $\dfrac{1}{2}E_F$ (d) E_F

ANSWER KEY

1. (a) **2. (d)** **3. (b)** **4. (d)** **5. (b)** **6. (a)** **7. (b)**
8. (d) **9. (d)** **10. (c)** **11. (b)** **12. (*)** **13. (a)** **14. (c)**

TIFR Previous Years' Questions

1. Consider a sealed but thermally conducting container of total volume V, Which is in equilibrium with a thermal bath at temperature T. The container is divided into two equal chambers by a thin but impermeable partition. One of these chambers contains an ideal gas, while the other half is a vacuum (see figure) **[TIFR 2012]**



If the partition is removed and the ideal gas is allowed to expand and fill the entire container, then the entropy per molecule of the system will increase by an amount

(a) $2k_B$ (b) $k_B\,\ell n(1/2)$ (c) $k_B\,\ell n2$ (d) $(k_B\,\ell n2)/2$

2. A system at temperature T has three energy states 0, $\pm\varepsilon$. The entropy of the system in the low temperature $(T\to 0)$ and high temperature $(T\to\infty)$ limits are, respectively, **[TIFR 2013]**

(a) $S_{T\to 0}=0$ and $S_{T\to\infty}=k_B\exp(-3)$ (b) $S_{T\to 0}=S_{T\to\infty}=k_B\ln 3$

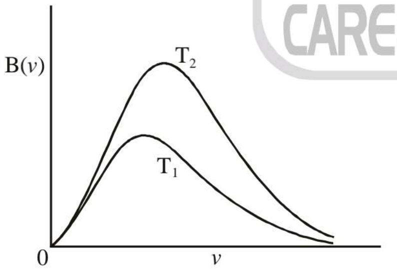
(c) $S_{T\to 0}=0$ and $S_{T\to\infty}=k_B\ln 3$ (d) $S_{T\to 0}=0$ and $S_{T\to\infty}=3k_B/2$

3. A gas of photons is enclosed in a container of fixed volume at an absolute temperature T. Noting that the photon is massless particle (i.e. its energy and momentum are related by E = pc), the number of photons in the container will vary as **[TIFR 2014]**

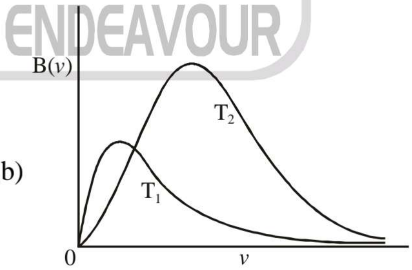
(a) T (b) T^2 (c) T^3 (d) T^4

4. Two blackbodies radiate energy at temperature T_1 and $T_2\,(T_1>T_2)$. The energy emitted per unit time per unit solid angle per unit surface area of a blackbody in the frequency range v to $v+dv$ is given by $B(v)dv$. Which one of the following graphs has the correct form ? **[TIFR 2015]**

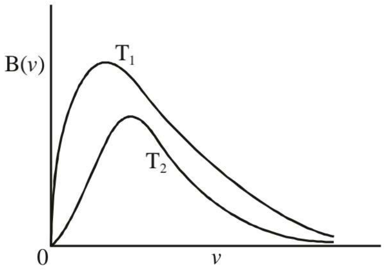
(a)



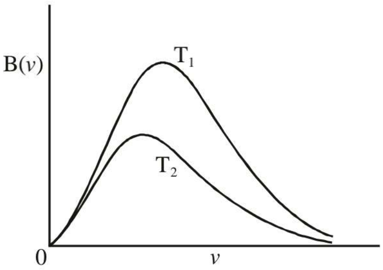
(b)



(c)



(d)



5. Bosonic excitations of ferromagnets have a dispersion relation $\in = \gamma k^2$, where $\in$ is the energy and k is the wavevector of the excitation. Assuming a system of such non-interacting bosonic excitation, at low temperature T, the specific heat C_V of a three dimensional ferromagnet will be proportional to **[TIFR 2015]**

(a) T (b) $T^{3/2}$ (c) $T^{5/2}$ (d) T^3

6. In two dimensions two metals A and B, have the number density of free electrons in the ratio $n_A : n_B = 1:2$.

The ratio of their Fermi energies is **[TIFR 2017]**

(a) $2:3$ (b) $1:2$ (c) $1:4$ (d) $1:8$

7. A one-dimensional quantum harmonic oscillator of natural frequency ω is in thermal equilibrium with a heat bath at temperature T. The mean value $\langle E \rangle$ of the energy of the oscillator can be written as

[TIFR 2017]

(a) $\frac{\hbar\omega}{2}\text{sech}\left(\frac{\hbar\omega}{2k_BT}\right)$ (b) $\frac{\hbar\omega}{2}\text{csch}\left(\frac{\hbar\omega}{2k_BT}\right)$ (c) $\frac{\hbar\omega}{2}\coth\left(\frac{\hbar\omega}{2k_BT}\right)$ (d) $\frac{\hbar\omega}{2}\tanh\left(\frac{\hbar\omega}{2k_BT}\right)$

8. Consider a thermal ensemble at temperature T which is composed to identical quantum harmonic oscillators of frequency ω_0 with non-overlapping wavefunctions. The probability that these will be an even number of energy quanta in the system is **[TIFR 2019]**

(a) $\tanh\left(\frac{\hbar\omega_0}{2k_BT}\right)$ (b) $\frac{1}{\exp\left(-\frac{\hbar\omega_0}{k_BT}\right)+1}$ (c) $\frac{1}{\exp\left(\frac{\hbar\omega_0}{k_BT}\right)+1}$ (d) $\frac{1}{\exp\left(-\frac{\hbar\omega_0}{k_BT}\right)-1}$

ANSWER KEY

1. (c) **2. (c)** **3. (c)** **4. (d)** **5. (b)** **6. (b)** **7. (c)**
8. (b)

Other Examinations Previous Years' Questions

1. The specific heat of an ideal Fermi gas in 3-dimension at very low temperatures (T) varies as
(a) T (b) $T^{3/2}$ (c) T^2 (d) T^3

2. The temperature of a cavity of fixed volume is doubled. Which of the following is true for the black-body radiation inside the cavity?
(a) Its energy and the number of photons both increase 8 times.
(b) Its energy increases 8 times and the number of photons increase 16 times.
(c) Its energy increases 16 times and the number of photons increases 8 times.
(d) Its energy and the number of photons both increases 16 times.

3. Consider free expansion of one mole of an ideal gas in an adiabatic container from volume V_1 to V_2. The entropy change of the gas, calculated by considering a reversible process between the original state (V_1, T) to the final state (V_2, T) where T is the temperature of the system, is denoted ΔS_1. The corresponding chagne in the entropy of the surrounding is ΔS_2. Which of the following combinations is correct?
(a) $\Delta S_1 = R\ln(V_1/V_2)$, $\Delta S_2 = -R\ln(V_1/V_2)$
(b) $\Delta S_1 = -R\ln(V_1/V_2)$, $\Delta S_2 = R\ln(V_1/V_2)$
(c) $\Delta S_1 = R\ln(V_2/V_1)$, $\Delta S_2 = 0$ (d) $\Delta S_1 = -R\ln(V_2/V_1)$, $\Delta S_2 = 0$

4. Let N_{MB}, N_{BE}, N_{FD} denote the number of ways in which two particles can be distributed in two energy states according to Maxwell-Boltzmann, Bose-Einstein and Fermi-Dirac statistics respectively. Then: $N_{MB} : N_{BE} : N_{FD}$ is:
(a) 4 : 3 : 1 (b) 4 : 2 : 3 (c) 4 : 3 : 3 (d) 4 : 3 : 2

5. A system consists of *N* number of particles, *N* >> 1. Each particle can have only one of the two energies E_1 or $E_1 + \varepsilon(\varepsilon > 0)$. If the system is in equilibrium at a temperature *T*, the average number of particles with energy E_1 is
(a) $\frac{N}{2}$ (b) $\frac{N}{e^{\varepsilon/kT}+1}$ (c) $\frac{N}{e^{-\varepsilon/kT}+1}$ (d) $Ne^{-\varepsilon/kT}$

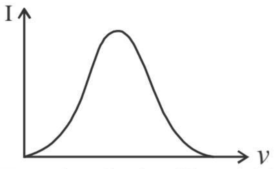
6. A rigid and thermally isolated tank is divided into two compartments of equal volume V, separated by a thin membrane. One compartment contains one mole of an ideal gas A and the other compartment contains one mole of a different ideal gas B. The two gases are in thermal equilibrium at a temperature T. If the membrane ruptures, the two gases mix. Assume that the gases are chemically inert. The change in the total entropy of the gases on mixing is
(a) 0 (b) $R\ln 2$ (c) $\frac{3}{2}R\ln 2$ (d) $2R\ln 2$

7. A system comprises of three electrons. There are three single particle energy levels accessible to each of these electrons. The number of possible configurations for this system is
(a) 1 (b) 3 (c) 6 (d) 7

8. The probability of a system to be in a state with energy E at temperature T is proportional to $e^{-E/\kappa_B T}$. If the system in question is a one dimensional quantum harmonic oscillator of frequency ω the average energy at temperature T is
(a) $\frac{\hbar\omega}{2}$ (b) $\hbar\omega\left(e^{-\hbar\omega/k_BT}+\frac{1}{2}\right)$ (c) $\hbar\omega\left(\frac{1}{e^{\hbar\omega/k_BT}-1}+\frac{1}{2}\right)$ (d) $\hbar\omega\left(\frac{1}{e^{\hbar\omega/k_BT}+1}+\frac{1}{2}\right)$

9. Let U, T, S and P denote, respectively, the internal energy, temperature, entropy and pressure of a thermodynamic system. Then a change ΔF in the free energy $F = U - TS$ measures.
(a) the heat exchanged at constant pressure (b) the work done by the system at constant entropy
(c) the work done on the system at constant temperature
(d) the heat exchanged at constant temperature.

10. The graph in the figure below shows the intensity I as a function of frequency v of a perfect blackbody at a fixed temperature T:



The corresponding graph at teperature 2T can be obtained by which of the following operations ?
For every point of the graph
(a) multiply the v-coordinate by $1/2$ and the I-coordinate by 8
(b) multiply the v-coordinate by 2 and the I-coordinate by 8
(c) multiply the v-coordinate by $1/2$ and the I-coordinate by 16
(d) multiply the v-coordinate by 2 and the I-coordinate by 16

11. A system in thermal equilibrium consists of subsystems A and B that interact only weakly with each other. If Z_A and Z_B are the canonical partition functions of A and B respectively, the partition function of the total system is given, to a good approximation, by
(a) $Z_A + Z_B$ (b) Z_A / Z_B (c) $Z_A - Z_B$ (d) $Z_A\ Z_B$

12. The standard deviation of the energy of a system in the canonical ensemble is equal to
(a) $\sqrt{C_v}$ (b) $k_B T^2 C_v$ (c) $\sqrt{k_B T^2 C_v}$ (d) $\sqrt{k_B T C_v}$

13. Let N be the total number of distinguishable particles in a system, out of which the number of particles with energies $E_1, E_2, ...E_n$ is $N_1, N_2............N_n$ respectively, $(N_1 + N_2 + ...N_n = N)$
The number of ways in which these particles can be distributed is
(a) $\dfrac{N!}{N_1!N_2!---N_n!}$ (b) $\dfrac{N_1!N_2!---N_n!}{N!}$
(c) $\dfrac{N!}{\sqrt{N_1!}+\sqrt{N_2!}---\sqrt{N_n!}}$ (d) $\dfrac{\sqrt{N!}}{N_1!N_2!---N_n!}$

14. The entropy of a system in the microcanonical ensemble formalism
(a) Is independent of the number of microslates.
(b) Is directly proportional to the number of microstates.
(c) Is inversely proportional to the number of microstates.
(d) Increases logarithmicaly with increase in number of microstates.

15. A degenerate electron gas is characterised by the following property:
(a) Each energy level has a very large occupancy.
(b) Each energy level has only a single occupancy.
(c) High density and low temperature.
(d) Low density and high temperature.

16. Z_1, Z_2 and Z_3 represent translational, vibrational and rotational partition functions of a diatomic molecule. The partition function of the molecule Z is given by
(a) $Z = Z_1\ Z_2\ Z_3$ (b) $Z = Z_1 + Z_2 + Z_3$
(c) $\ell n\, Z = \ell n\left(Z_1 + Z_2 + Z_3\right)$ (d) $e^Z = e^{Z_1} + e^{Z_2} + e^{Z_3}$

17. The principle of equal a priori probability asserts than
(a) All states of a system are equally probable.
(b) All microstates of a system compatible with a macrostate of energy between E and $E + \Delta E$ are equally probable.
(c) All microstates of a system which are incompatible with a macrostate energy E are equally probable.
(d) A-priori probability of all microstates has a normal distribution.

18. In a statistical description of a system in the canonical ensemble, the relative uncertainty in the energy of the system, $\Delta E/E$, varies with the number of particles N in the systems as
(a) N^{-1} (b) $N^{-1/2}$ (c) N^{0} (d) $N^{1/2}$

19. In a canonical ensemble description of a system, which of the following quantities remain fixed?
(a) Energy of the system (b) Square of the energy of the system
(c) Volume of the system (d) Number of particles in the system.

20. A system has two energy levels 0 and $\varepsilon > 0$. The energy levels are populated according to the Boltzmann's distribution law. In the limit $T \to \infty$ the entropy per particle is:
(a) $+\infty$ (b) $k_B \ln 2$ (c) $2k_B$ (d) $k_B \ln e^2$

21. Consider a system with two allowed microstates, one at energy zero and the other at energy $k_B T$ The system is in equilibrium at temperature T. What is the probability of finding the system in its ground state?
(a) 1 (b) $\frac{1}{e}$ (c) $\frac{1}{1+e}$ (d) $\frac{e}{1+e}$

22. In the canonical ensemble description, a system
(a) can only exchange energy with its environment
(b) is isolated from its environment
(c) can exchange energy as well as particles with its environment
(d) can only exchange particles with its environment

23. A gas of N spin 1/2 particles and magnetic moment $\vec{\mu}$ each, is kept in uniform magnetic induction $\vec{B}$ at temperature T. Then, average energy of the gas is:
(a) $N\mu B\, e^{-\beta\mu B}$ (b) $-N\mu B\, e^{\beta\mu B}$
(c) $N\mu B\left(e^{\beta\mu B} + e^{-\beta\mu B}\right)$ (d) $\frac{-N\mu B\left(e^{\beta\mu B} - e^{\beta\mu B}\right)}{\left(e^{-\beta\mu B} + e^{\beta\mu B}\right)}$
Where $\beta = 1/k_B T$; k_B is Boltzmann constant.

24. A system of particles can undergo a Bose Einstein condensation if all particles
(a) Have zero rest mass and zero spin (b) Have half-integer spins
(c) Have mass and spin zero (d) Are free electrons.

25. A particle of spin $\frac{1}{2}$ and magnetic moment μ is in a magnetic field B. Its partition function is:
(a) $2 \tanh \mu B/kT$ (b) $2\cosh \frac{\mu B}{kT}$ (c) $2 \sin h \frac{\mu B}{kT}$ (d) $e^{\frac{\mu B}{kT}}$

26. A system consists of two Bosons each of which can occupy two energy levels 0 and E_0. The partition function for the system is:
(a) $1 + e^{-\beta E_0}$ (b) $1 + e^{-2\beta E_0}$ (c) $1 + e^{-\beta E_0} + e^{-2\beta E_0}$ (d) $e^{-\beta E_0} + e^{-2\beta E_0}$, Where $\beta = \frac{1}{kT}$.

27. According to Bose-Einstein Statistics, there exists a 'Bose Condensate' which is a collections of bosons. For this condense, which of the following statements is correct?

(a) As $T \to \infty$, all bosons reside in excited states

(b) For $T < T_c$ almost all bosons reside in the ground state.

(c) As $T \to \infty$, all bosons reside in the ground state.

(d) For $T > T_c$, bosons behave similar to fermions.

28. Plank's distribution formula black body radiation is given by

$$u_\lambda d_\lambda = \frac{8\pi hc}{-\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT}-1}$$

For long wave lengths and high temperatures, u_λ, d_λ becomes equal to

(a) $\frac{8\pi kT}{\lambda^4}d\lambda$ (b) $\frac{8\pi kT}{\lambda^5}d\lambda$ (c) $\frac{8\pi kT}{\lambda^5}e^{hc/\lambda kT}$ (d) $\frac{8\pi kT}{\lambda^5}\left(e^{-h/\lambda kT}+1\right)$

29. The number of ways in which 5 identical bosons can be distributed in 4 states is:

(a) $\frac{8!}{5!3!}$ (b) $\frac{9!}{5!4!}$ (c) $\frac{9!}{4!4!}$ (d) $\frac{8!}{4!4!}$

30. Blackbody radiation is enclosed inside a spherical cavity of radius r at a temperature T. What would be the temperature of the enclosure if the radius expands to 2*r* adiabatically?

(a) T/2 (b) T (c) $T/\sqrt{2}$ (d) 2T

ANSWER KEY

1. (a) **2. (c)** **3. (c)** **4. (a)** **5. (c)** **6. (d)** **7. (d)**
8. (c) **9. (c)** **10. (d)** **11. (d)** **12. (c)** **13. (a)** **14. (d)**
15. (c) **16. (a)** **17. (b)** **18. (b)** **19. (d)** **20. (b)** **21. (d)**
22. (a) **23. (d)** **24. (a,c)** **25. (b)** **26. (c)** **27. (a)** **28. (a)**
29. (a) **30. (a)**

GATE Previous Years' Questions

Common Data for Q.1 and Q.2.

A gas of N particles is enclosed in a volume V at a temperature T. The logarithm of the partition function is given by $\ln Z = N\ln\left[(V-bN)(k_BT)^{3/2}\right]$ where b is a constant with appropriate dimensions.

1. If P is the pressure of the gas, the equation of state is given by **[GATE 2003]**

(a) $P(V-bN) = Nk_BT$ (b) $P(V-bN) = k_BT$

(c) $P(V-b) = Nk_BT$ (d) $P(V-b) = Nk_BT$

2. The internal energy of the gas is given by **[GATE 2003]**

(a) $U=(1/2)k_BT$ (b) $U=(1/2)k_BT$ (c) $U=(3/2)Nk_BT$ (d) $U=2Nk_BT$

Common data for Q.3, Q.4 and Q.5.

A system consists of three spin-half particles, the z components of whose spins $S_z(1), S_z(2)$ and $S_z(3)$ can take value $+1/2$ and $-1/2$. The total spin of the system is $S_z = S_z(1)+S_z(2)+S_z(3)$ **[GATE 2003]**

3. The total number of possible micro-states of this system is:

(a) 3 (b) 6 (c) 7 (d) 8

4. The total number of micro-states with $S_z = 1/2$ is:

(a) 3 (b) 5 (c) 6 (d) 7

5. Consider an ensemble of systems where each microstate has equal probability. The ensemble average of S_z is:

(a) –1/2 (b) 0 (c) 1/2 (d) 3/2

Common Data for Q.6 and Q.7.

An ensemble of N three-level systems with energies $E = -\epsilon_0, 0, +\epsilon_0$ is in thermal equilibrium at temperature T. Let $\beta = (k_\beta T)^{-1}$ **[GATE 2004]**

6. If $\beta\varepsilon_0 = 2$, the probability of finding the system in the level $\varepsilon = 0$ is:

(a) $(\cos h\,2)/2$ (b) $(\cos h\,2)^{-1}$ (c) $(2\cos h\,2)^{-1}$ (d) $(1+2\cos h\,2)^{-1}$

7. The free energy of the system at high temperature $(\text{i.e., } x \equiv \beta\varepsilon_0 << 1)$ is approximately

(a) $-NK_B\,Tx^2$ (b) $-NK_B\,T\left[\ln 2 + x^2/2\right]$

(c) $-NK_BT\left[\ln 3 + x^2/3\right]$ (d) $-NK_BT\ln 3$

Common Data for Q.8 and Q.9.

Consider a cavity of volume V at temperature T **[GATE 2006]**

8. The density of states at energy E of the quantized radiation (photons) is:

(a) $\frac{8\pi V}{h^3C^3}E^2$ (b) $\frac{8\pi V}{h^3C^3}E^{3/2}$ (c) $\frac{8\pi V}{h^3C^3}E$ (d) $\frac{8\pi V}{h^3C^3}E^{1/2}$

9. The average number of photons in equilibrium inside the cavity is proportional to

(a) T (b) T^2 (c) T^3 (d) T^4.

Common Data for Q. 10 and Q. 11

An ensemble of quantum oscillator is kept at a finite temperature $T = \frac{1}{k_B \beta}$

10. The partition function of a single oscillator with energy levels $\left(n+\frac{1}{2}\right)\hbar\omega$ is given by **[GATE 2007]**

(a) $Z = \frac{e^{-\beta\hbar\omega/2}}{1-e^{-\beta\hbar\omega}}$ (b) $Z = \frac{e^{-\beta\hbar\omega/2}}{1+e^{-\beta\hbar\omega}}$ (c) $Z = \frac{1}{1-e^{-\beta\hbar\omega}}$ (d) $Z = \frac{1}{1+e^{-\beta\hbar\omega}}$

11. The average number of energy quanta of the oscillators is given by **[GATE 2007]**

(a) $\langle n \rangle = \frac{1}{e^{\beta\hbar\omega}-1}$ (b) $\langle n \rangle = \frac{e^{-\beta\hbar\omega}}{e^{\beta\hbar\omega}-1}$ (c) $\langle n \rangle = \frac{1}{e^{\beta\hbar\omega}+1}$ (d) $\langle n \rangle = \frac{e^{-\beta\hbar\omega}}{e^{\beta\hbar\omega}+1}$

Common data for Q. 12 and Q. 13.

Consider two dimentional electron gas of N electrons of mass 'm' each in a system of size L×L.

12. The density of states between energy ε and $\varepsilon + d\varepsilon$ is **[GATE 2008]**

(a) $\frac{4\pi L^2 m}{h^2} d\varepsilon$ (b) $\frac{4\pi L^2 m}{h^2} \frac{1}{\sqrt{\varepsilon}} d\varepsilon$ (c) $\frac{4\pi L^2 m}{h^2} \sqrt{\varepsilon}\, d\varepsilon$ (d) $\frac{4\pi L^2 m}{h^2} \varepsilon\, d\varepsilon$

13. The ground state energy E_0 of the system in terms of the Fermi energy E_F and the number of electrons N is given by **[GATE 2008]**

(a) $\frac{1}{3} NE_F$ (b) $\frac{1}{2} NE_F$ (c) $\frac{2}{3} NE_F$ (d) $\frac{3}{5} NE_F$

Common data for Q. 14 and Q. 15.

Consider a two level quantum system with energies $\epsilon_1 = 0$ and $\epsilon_2 = \epsilon$

14. The Helmholtz free energy of the system is given by **[GATE 2009]**

(a) $-k_B T \ln\left(1+e^{-\varepsilon/k_B T}\right)$ (b) $k_B T \ln\left(1+e^{-\varepsilon/k_B T}\right)$

(c) $\frac{3}{2} k_B T$ (d) $\varepsilon - k_B T$

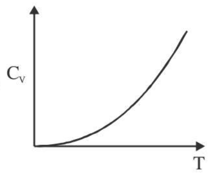
15. The specific heat of the system is given **[GATE 2009]**

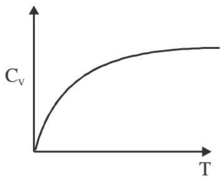
(a) $\frac{\varepsilon}{k_B T} \frac{e^{-\varepsilon/k_B T}}{\left(1+e^{-\varepsilon/k_B T}\right)^2}$ (b) $\frac{\varepsilon^2}{k_B T^2} \frac{e^{-\varepsilon/k_B T}}{\left(1+e^{-\varepsilon/k_B T}\right)}$ (c) $-\frac{\varepsilon^2\, e^{-\varepsilon/k_B T}}{\left(1+e^{-\varepsilon/k_B T}\right)^2}$ (d) $\frac{\varepsilon^2}{k_B T^2} \frac{e^{-\varepsilon/k_B T}}{\left(1+e^{-\varepsilon/k_B T}\right)^2}$

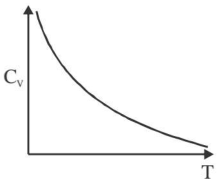
Common Data for Q . 16 and Q. 17

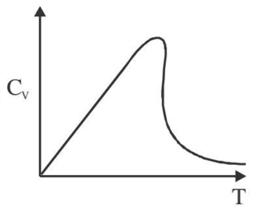
The partition function for a gas photons is given by $\ln Z = \frac{\pi^2 V \left(k_R T\right)^3}{45\, \hbar^3 C^3}$

16. The specific heat of photon gas varies with temperature as **[GATE 2010]**

(a) 

(b) 

(c) 

(d) 

17. The pressure of the photon gas is **[GATE 2010]**

(a) $\frac{\pi^2}{15}\frac{(k_BT)^3}{\hbar^3c^3}$ (b) $\frac{\pi^2}{8}\frac{(k_BT)^4}{\hbar^3c^3}$ (c) $\frac{\pi^2}{45}\frac{(k_BT)^4}{\hbar^3c^3}$ (d) $\frac{\pi^2}{45}\frac{(k_BT)^{3/2}}{\hbar^3c^3}$

18. A system has two energy levels with energies ε and 2ε. The lower level is 4-fold degenerate while the upper level is doubly degenerate. If there are N non-interacting classical particles in the system, which is in thermodynamic equilibrium at a temperature T, the fraction of particles in the upper level is: **[GATE 2011]**

(a) $\frac{1}{1+e^{-\varepsilon/k_BT}}$ (b) $\frac{1}{1+2e^{\varepsilon/k_BT}}$ (c) $\frac{1}{2e^{\varepsilon/k_BT}+4e^{2\varepsilon/k_BT}}$ (d) $\frac{1}{2e^{\varepsilon/k_BT}-4e^{2\varepsilon/k_BT}}$

19. Consider a system whose three energy levels are given by 0, ε and 2ε. The energy level ε is two-fold degenerate and the other two are non-degenerate. The partition function of the system with $\beta=\frac{1}{k_BT}$ is given by **[GATE 2012]**

(a) $1+2e^{-\beta\in}$ (b) $2e^{-\beta\in}+e^{-2\beta\in}$ (c) $\left(1+e^{-\beta\in}\right)^2$ (d) $1+e^{-\beta\in}+e^{-2\beta\in}$

20. A paramagnetic system consisting of N spin-half particles, is placed in an external magnetic field. It is found that N/2spins are aligned parallel and the remaining N/2 spins are aligned antiparallel to the magnetic field. The statistical entropy of the system is, **[GATE 2012]**

(a) $2Nk_B\ell n\,2$ (b) $\frac{N}{2}k_B\,\ell n\,2$ (c) $\frac{3N}{2}k_B\,\ell n\,2$ (d) $Nk_B\,\ell n\,2$

Common Data for Q. 21 and Q. 22:

There are four energy levels E, $2E$, $3E$ and $4E$ (where $E > 0$). The canonical partition function of two particles is, if these particles are **[GATE 2013]**

21. Two identical fermions

(a) $e^{-2\beta E}+e^{-4\beta E}+e^{-6\beta E}+e^{-8\beta E}$ (b) $e^{-3\beta E}+e^{-4\beta E}+2e^{-5\beta E}+e^{-6\beta E}+e^{-7\beta E}$

(c) $(e^{-\beta E}+e^{-2\beta E}+e^{-3\beta E}+e^{-4\beta E})^2$ (d) $e^{-2\beta E}-e^{-4\beta E}+e^{-6\beta E}-e^{-8\beta E}$

22. Two distinguishable particles

(a) $e^{-2\beta E}+e^{-4\beta E}+e^{-6\beta E}+e^{-8\beta E}$ (b) $e^{-3\beta E}+e^{-4\beta E}+2e^{-5\beta E}+e^{-6\beta E}+e^{-7\beta E}$

(c) $(e^{-\beta E}+e^{-2\beta E}+e^{-3\beta E}+e^{-4\beta E})^2$ (d) $e^{-2\beta E}-e^{-4\beta E}+e^{-6\beta E}-e^{-8\beta E}$

23. For a system of two bosons, each of which can occupy any of the two energy levels 0 and ε, the mean energy of the system at a temperature T with $\beta=\frac{1}{k_BT}$ is given by **[GATE 2014]**

(a) $\frac{\varepsilon\, e^{-\beta\varepsilon}+2\varepsilon\, e^{-2\beta\varepsilon}}{1+2e^{-\beta\varepsilon}+e^{-2\beta\varepsilon}}$ (b) $\frac{1+\varepsilon e^{-\beta\varepsilon}}{2e^{-\beta\varepsilon}+e^{-2\beta\varepsilon}}$ (c) $\frac{2\varepsilon\, e^{-\beta\varepsilon}+\varepsilon\, e^{-2\beta\varepsilon}}{2+e^{-\beta\varepsilon}+e^{-2\beta\varepsilon}}$ (d) $\frac{\varepsilon\, e^{-\beta\varepsilon}+2\varepsilon\, e^{-2\beta\varepsilon}}{2+e^{-\beta\varepsilon}+e^{-2\beta\varepsilon}}$

24. The average energy U of a one dimensional quantum oscillator of frequency ω and in contact with a heat bath at temperature T is given by **[GATE 2015]**

(a) $U=\frac{1}{2}\hbar\omega\coth\left(\frac{1}{2}\beta\hbar\omega\right)$ (b) $U=\frac{1}{2}\hbar\omega\sinh\left(\frac{1}{2}\beta\hbar\omega\right)$

(c) $U=\frac{1}{2}\hbar\omega\tanh\left(\frac{1}{2}\beta\hbar\omega\right)$ (d) $U=\frac{1}{2}\hbar\omega\cosh\left(\frac{1}{2}\beta\hbar\omega\right)$

25. Consider a system of N non-interacting spin-$\frac{1}{2}$ particles, each having a magnetic moment μ, is in a magnetic field $\vec{B} = B\,\hat{z}$. If E is the total energy of the system, the number of accessible microstates Ω is given by

(a) $\Omega = \dfrac{N!}{\frac{1}{2}\left(N - \frac{E}{\mu B}\right)!\;\frac{1}{2}\left(N + \frac{E}{\mu B}\right)!}$ (b) $\Omega = \dfrac{\left(N - \frac{E}{\mu B}\right)!}{\left(N + \frac{E}{\mu B}\right)!}$ **[GATE 2015]**

(c) $\Omega = \frac{1}{2}\left(N - \frac{E}{\mu B}\right)!\;\frac{1}{2}\left(N + \frac{E}{\mu B}\right)!$ (d) $\Omega = \dfrac{N!}{\left(N + \frac{E}{\mu B}\right)!}$

26. The entropy of a gas containing N particles enclosed in a volume V is given by $S = Nk_B\,\ell n\left(\dfrac{aVE^{3/2}}{N^{5/2}}\right)$, where E is the total energy, a is a constant and k_B is the Boltzmann constant. The chemical potential μ of the system at a temperature T is given by **[GATE 2015]**

(a) $\mu = -k_BT\left[\ell n\left(\dfrac{aVE^{3/2}}{N^{5/2}}\right) - \dfrac{5}{2}\right]$ (b) $\mu = -k_BT\left[\ell n\left(\dfrac{aVE^{3/2}}{N^{5/2}}\right) - \dfrac{3}{2}\right]$

(c) $\mu = -k_BT\left[\ell n\left(\dfrac{aVE^{3/2}}{N^{3/2}}\right) - \dfrac{5}{2}\right]$ (d) $\mu = -k_BT\left[\ell n\left(\dfrac{aVE^{3/2}}{N^{3/2}}\right) - \dfrac{3}{2}\right]$

27. The entropy S of a system of N spins, which may align either in the upward or in the downward direction, is given by $S = -k_BN\left[p\ln p + (1-p)\ln(1-p)\right]$. Here, k_B is the Boltzmann constant. The probability of alignment in the upward direction is p. The value of p, at which the entropy is maximum, is ______________. (Give your answer upto one decimal place) **[GATE 2016]**

28. Consider a system having three energy levels with energies $0, 2\varepsilon$ and 3ε, with respective degeneracies of 2, 2 and 3. Four bosons of spin zero have to be accommodated in these levels such that the total energy of the system is 10ε. The number of ways in which it can be done is ________ **[GATE 2016]**

29. Three particles are to be distributed in four non-degenerate energy levels. The possible number of ways of distribution: (i) for distinguishable particles, and (ii) for identical Bosons, respectively, is
(a) (i) 24, (ii) 4 (b) (i) 24, (ii) 20 (c) (i) 64, (ii) 20 (d) (i) 64, (ii) 16 **[GATE 2018]**

30. The partition function of an ensemble at a temperature T is

$$Z = \left(2\cosh\frac{\varepsilon}{k_BT}\right)^N,$$

where k_B is the Boltzmann constant. The heat capacity of this ensemble at $T = \dfrac{\varepsilon}{k_B}$ is $X\,Nk_B$, where the value of X is ________ (up to two decimal places). **[GATE 2018]**

31. The quantum effects in an ideal gas become important below a certain temperature T_Q when de-Broglie wavelength corresponding to the root mean square thermal speed becomes equal to the inter-atomic separation. For such a gas of atoms of mass 2×10^{-26} kg and number density $6.4\times10^{25}\ \mathrm{m}^{-3}$, $T_Q =$ ________ $\times 10^{-3}$ K (up to one decimal place). **[GATE 2018]**

32. A microcanonical ensemble consists of 12 atoms with each taking either energy 0 state, or energy ε state. Both states are non-degenerate. If the total energy of this ensemble is 4ε, its entropy will be ________ k_B (up to one decimal place), where k_B is the Boltzmann constant. **[GATE 2018]**

33. A large number *N* of ideal bosons, each of mass *m*, are trapped in a three-dimensional potential

$$V(r)=\frac{m\omega^2r^2}{2}$$

The bosonic system is kept at temperature *T* which is much lower than the Bose-Einstein condensation temperature T_c. The chemical potential (μ) satisfies **[GATE 2019]**

(a) $\mu\le\frac{3}{2}\hbar\omega$ (b) $2\hbar\omega>\mu>\frac{3}{2}\hbar\omega$ (c) $3\hbar\omega>\mu>2\hbar\omega$ (d) $\mu=3\hbar\omega$

34. Consider two systems A and B each having two distinguishable particles. In both the systems, each particle can exist in states with energies 0, 1, 2 and 3 units with equal probability. The total energy of the combined system is 5 units. Assuming that the system A has energy 3 units and the system B has energy 2 units, the entropy of the combined system is $k_B\ln\lambda$. The value of λ is _________. **[GATE 2019]**

ANSWER KEY

1. (a) **2. (c)** **3. (d)** **4. (a)** **5. (b)** **6. (d)** **7. (d)**
8. (a) **9. (c)** **10. (a)** **11. (a)** **12. (a)** **13. (b)** **14. (a)**
15. (d) **16. (a)** **17. (c)** **18. (b)** **19. (c)** **20. (d)** **21. (b)**
22. (c) **23. (a)** **24. (a)** **25. (a)** **26. (a)** **27. (0.5)** **28. (18)**
29. (c) **30. (0.41 to 0.43)** **31. (78 to 90)** **32. (6.1 to 6.3)**
33. (a) **34. (12)**

CSIR-UGC-NET Previous Years' Questions

1. Consider a system of N non-interacting spins, each of which has classifical magnetic moment of magnitude μ. The Hamiltonian of this system in an external magnetic field $\vec{H}$ is $\mathcal{H} = -\sum_{i=1}^{N} \vec{\mu}_i \cdot \vec{H}$, where $\vec{\mu}_i$ is the magnetic moment of the ith spin. The magnetization per spin at temperature T is: **[CSIR June 2011]**

(a) $\frac{\mu^2 H}{k_B T}$ (b) $\mu\left[\coth\left(\frac{\mu H}{k_B T}\right) - \frac{k_B T}{\mu H}\right]$

(c) $\mu \sinh\left(\frac{\mu H}{k_B T}\right)$ (d) $\mu \tanh\left(\frac{\mu H}{k_B T}\right)$

2. Consider two independently diffusing non- interacting particles in 3-dimensional space, both placed at the origin at time t = 0. These particles have different diffusion constants D_1 and D_2. The quantity $\left\langle\left[\vec{R}_1(t) - \vec{R}_2(t)\right]^2\right\rangle$ where $\vec{R}_1(t)$ and $\vec{R}_2(t)$ are the positions of the particles at time t, behaves as:

(a) $6t(D_1 + D_2)$ (b) $6t(D_1 - D_2)$ (c) $6t\sqrt{D_1^2 + D_2^2}$ (d) $6t\sqrt{D_1 D_2}$ **[NET June 2011]**

3. A gas of N non-interacting particles is in thermal equilibrium at temperature T. Each particle can be in any of the possible non-degenerate states of energy 0, 2ε and 4ε. The average energy per particle of the gas, when $\beta\varepsilon \ll 1$, is: **[CSIR Dec. 2011]**

(a) 2ε (b) 3ε (c) $2\varepsilon/3$ (d) ε

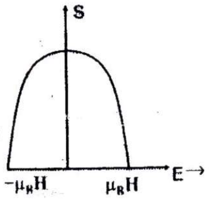
4. A one-dimensional chain consists of a set of N rods each of length a. When stretched by a load, each rod can align either parallel or perpendicular to the length of the chain. The energy of a rod is $-\varepsilon$ when aligned parallel to the length of the chain and is $+\varepsilon$ when perpendicular to it. When the chain is in thermal equilibrium at temperature T, its average length is: **[CSIR Dec. 2011]**

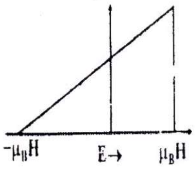
(a) Na/2 (b) Na (c) $Na/\left(1+e^{-2\varepsilon/k_B T}\right)$ (d) $Na/\left(1+e^{-2\varepsilon/k_B T}\right)$

5. Gas molecules of mass 'm' are confined in a cylinder of radius R and height L (with $R \gg L$) kept vertically in the Earth's gravitational field . The average energy of the gas at low temperatures (such that $mgL >> k_B T$) is given by **[NET Dec. 2011]**

(a) $Nk_BT/2$ (b) $3Nk_BT/2$ (c) $2Nk_BT$ (d) $5Nk_BT/2$

6. A system of N noninteracting spin $-\frac{1}{2}$ particles is placed in an external magnetic field H. The behaviour of the entropy of the system as a function of energy is given by **[CSIR Dec. 2011]**

(a)


(b)


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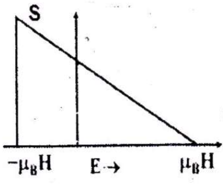

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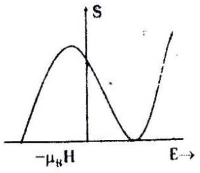


7. A system has two normal modes of vibration, with frequencies ω_1 and $\omega_2 = 2\omega_1$. What is the probability that at temperature T, the system has an energy less than $4\hbar\omega_1$?[In the following $x = e^{-\beta\hbar\omega}$ and Z is the partition function] **[CSIR June 2012]**

(a) $x^{3/2}\left(x+2x^2\right)/Z$
(b) $x^{3/2}\left(1+x+x^2\right)/Z$
(c) $x^{3/2}\left(1+2x^2\right)/Z$
(d) $x^{3/2}\left(1+x+2x^2\right)/Z$

8. Bose condensation occurs in liquid He^4 kept at ambient pressure at 2.17 K. At which temperature will Bose condensation occur in He^4 in gaseous state, the density of which is 1000 times smaller than that of liquid He^4? (Assume that it is a perfect Bose gas) **[CSIR June 2012]**

(a) 2.17 mK (b) 21.7 mK (c) 21.7 μK (d) 2.17 μK

9. Consider a one-dimensional Ising model with N spins, at very low temperatures when almost all the spins are aligned parallel to each other. There will be a few spin flips with each flip costing an energy 2J. In a configuration with *r* spin flips, the energy of the system is $E = -NJ + 2rJ$ and the number of configuration is ${}^{N}C_r$; *r* varies from 0 to N. The partition function is **[CSIR Dec. 2012]**

(a) $\left(\dfrac{J}{k_BT}\right)^N$ (b) e^{-NJ/k_BT} (c) $\left(\sinh\dfrac{J}{k_BT}\right)^N$ (d) $\left(\cosh\dfrac{J}{k_BT}\right)^N$

10. Non-interacting bosons undergo Bose-Einstein Condensation (BEC) when trapped in a three-dimensional isotropic simple harmonic potential. For BEC to occur, the chemical potential must be equal to

(a) $\hbar\omega/2$ (b) $\hbar\omega$ (c) $3\hbar\omega/2$ (d) 0 **[CSIR Dec. 2012]**

11. Consider a system of three spins S_1, S_2 and S_3 each of which can take values +1 and –1. The energy of the system is given by $E = -J\left[S_1S_2 + S_2S_3 + S_3S_1\right]$, where J is a positive constant. The minimum energy and the corresponding number of spin configurations are, respectively, **[CSIR Dec. 2012]**

(a) J and 1 (b) –3J and 1 (c) –3J and 2 (d) –6J and 2

12. Consider a system of two Ising spins S_1 and S_2 taking value ± 1 with interaction energy given by $\varepsilon = -JS_1S_2$, when it is in thermal equilibrium at temperature T. For large T, the average energy of the system varies as C/k_BT, with C given by **[CSIR June 2013]**

(a) $-2J^2$ (b) $-J^2$ (c) J^2 (d) 4J

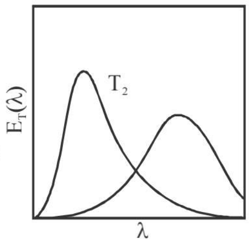
13. Consider two different systems each with three identical non-interacting particles. Both have single particle states with energies ε_0, $3\varepsilon_0$ and $5\varepsilon_0$, ($\varepsilon_0 > 0$). One system is populated by spin-$\frac{1}{2}$ fermions and the other by bosons. What is the value of $E_F - E_B$ where E_F and E_B are the ground state energies of the fermionic and bosonic systems respectively? **[CSIR June 2013]**

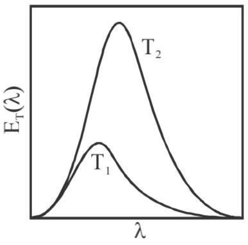
(a) $6\varepsilon_0$ (b) $2\varepsilon_0$ (c) $4\varepsilon_0$ (d) ε_0

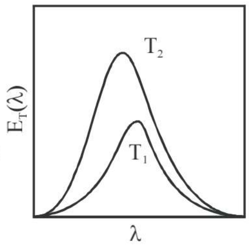
14. Three identical spin $-\frac{1}{2}$ fermions are to be distributed in two non-degenerate distinct energy levels. The number of ways this can be done is **[CSIR Dec. 2013]**

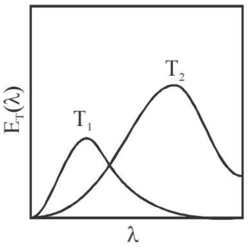
(a) 8 (b) 4 (c) 3 (d) 2

15. Which of the graphs below gives the correct qualitative behavior of the energy density $E_T(\lambda)$ of blackbody radiation of wavelength λ at two temperatures T_1 and T_2 ($T_1 < T_2$)? **[CSIR June 2014]**

(a)


(b)


(c)


(d)


16. A system can have three energy levels:
$E = 0, \pm\varepsilon$. The level E = 0 is doubley degenerate, while the others are non-degenerate. The average energy at inverse temperature β is **[CSIR June 2014]**

(a) $-\varepsilon\tanh(\beta\varepsilon)$ (b) $\dfrac{\varepsilon\left(e^{\beta\varepsilon}-e^{-\beta\varepsilon}\right)}{\left(1+e^{\beta\varepsilon}+e^{-\beta\varepsilon}\right)}$ (c) Zero (d) $-\varepsilon\tanh\left(\dfrac{\beta\varepsilon}{2}\right)$

17. The free energy F of a system depends on a thermodynamics variable ψ as **[NET June 2014]**

$$F = -a\psi^2 + b\psi^6$$

with a, b > 0. The value of ψ, when the system is in thermodynamic equilibrium, is

(a) zero (b) $\pm(a/6b)^{1/4}$ (c) $\pm(a/3b)^{1/4}$ (d) $\pm(a/b)^{1/4}$

18. For a particular thermodynamics system the entropy S is related to the internal energy U and volume V by

$$S = c\,U^{3/4}\,V^{1/4}$$

where *c* is a constant. The Gibbs potential $G = U - TS + pV$ for this system is **[CSIR June 2014]**

(a) $\dfrac{3pU}{4T}$ (b) $\dfrac{cU}{3}$ (c) zero (d) $\dfrac{US}{4V}$

19. The average local internal magnetic field acting on an Ising spin is $H_{\text{int}} = \alpha M$, where M is the magnetization and α is a positive constant. At a temperature T sufficiently close to (and above) the critical temperature T_c, the magnetic susceptibility at zero external field is proportional to (k_B is the Boltzmann constant) **[CSIR June 2014]**

(a) $k_BT - \alpha$ (b) $(k_BT + \alpha)^{-1}$ (c) $(k_BT - \alpha)^{-1}$ (d) $\tanh(k_BT + \alpha)$

20. The van der Waals equation of state for a gas is given by $\left(P + \dfrac{a}{V^2}\right)(V - b) = RT$ where, P, V and T represent the pressure, volume and temperature respectively, and a and b are constant parameters. At the critical point, where all the roots of the above cubic equation are degenerate, the volume is given by **[CSIR June 2014]**

(a) $\dfrac{a}{9b}$ (b) $\dfrac{a}{27b^2}$ (c) $\dfrac{8a}{27bR}$ (d) $3b$

21. The Hamiltonian of a classical particle moving in one dimension is $H=\frac{p^2}{2m}+\alpha q^4$ where α is a positive constant and p and q are its momentum and position respectively. Given that its total energy $E\le E_0$ the available volume of phase space depends on E_0 as **[CSIR Dec. 2014]**

(a) $E_0^{3/4}$ (b) E_0 (c) $\sqrt{E_0}$ (d) is independent of E_0

22. An ideal Bose gas is confined inside a container that is connected to a particle reservoir. Each particle can occupy a discrete set of single-particle quantum states. If the probability that a particular quantum state is unoccupied is 0.1, then the average number of bosons in that state is **[CSIR Dec. 2014]**

(a) 8 (b) 9 (c) 10 (d) 11

23. A collection N of non-interacting spins S_i, $i=1,2,...,N,(S_i=\pm 1)$ is kept in an external magnetic field B at a temperature T. The Hamiltonian of the system is $H=-\mu B\sum_i S_i$. What should be the minimum value of $\frac{\mu B}{k_BT}$ for which the mean value $\langle S_i\rangle\ge\frac{1}{3}$? **[CSIR Dec. 2014]**

(a) $\frac{1}{2}N\ln 2$ (b) $2\ln 2$ (c) $\frac{1}{2}\ln 2$ (d) $N\ln 2$

24. A random walker takes a step of unit length in the positive direction with probability 2/3 and a step of unit length in the negative direction with probability 1/3. The mean displacement of the walker after n steps is

(a) $n/3$ (b) $n/8$ (c) $2n/3$ (d) 0 **[CSIR Dec. 2014]**

25. A system of N non-intersecting classical particles, each of mass m is in a two-dimensional harmonic potential of the form $V(r)=\alpha(x^2+y^2)$ where α is a positive constant. The canonical partition function of the system at temperature T is $\left(\beta=\frac{1}{k_BT}\right)$. **[CSIR June 2015]**

(a) $\left[\left(\frac{\alpha}{2m}\right)^2\frac{\pi}{\beta}\right]^N$ (b) $\left(\frac{2m\pi}{\alpha\beta}\right)^{2N}$ (c) $\left(\frac{\alpha\pi}{2m\beta}\right)^N$ (d) $\left(\frac{2m\pi^2}{\alpha\beta^2}\right)^N$

26. Consider three Ising spins at the vertices of a triangle which interact with each other with a ferromagnetic Ising interaction of strength J. The partition function of the system at temperature T is given by $\left(\beta=\frac{1}{k_BT}\right)$:

(a) $2e^{3\beta J}+6e^{-\beta J}$ (b) $2e^{-3\beta J}+6e^{\beta J}$ **[CSIR June 2015]**

(c) $2e^{3\beta J}+6e^{-3\beta J}+3e^{\beta J}+3e^{-\beta J}$ (d) $(2\cosh\beta J)^3$

27. An ideal Bose gas in d-dimensions obeys the dispersion relation $\varepsilon(\hat{k})=Ak^s$, where A and s are constants. For Bose-Einstein condensation to occur, the occupancy of excited states **[CSIR June 2015]**

$$N_e=c\int_0^\infty\frac{\varepsilon^{(d-s)/s}}{e^{\beta(\varepsilon-\mu)}-1}d\varepsilon$$

where c is a constant, should remain finite even for $\mu=0$. This can happen if

(a) $\frac{d}{s}<\frac{1}{4}$ (b) $\frac{1}{4}<\frac{d}{s}<\frac{1}{2}$ (c) $\frac{d}{s}>1$ (d) $\frac{1}{2}<\frac{d}{s}<1$

28. The Hamiltonian of a system of N non-interacting spin-$\frac{1}{2}$ particles is $H=-\mu_0 B\sum_i S_i^z$, where $S_i^z=\pm 1$ are the components of i^{th} spin along an external magnetic field B. At a temperature T such that $e^{\mu_0 B/k_BT}=2$, the specific heat per particle is **[CSIR Dec. 2015]**

(a) $\frac{16}{25}k_B$ (b) $\frac{8}{25}k_B\ln 2$ (c) $k_B(\ln 2)^2$ (d) $\frac{16}{25}k_B(\ln 2)^2$

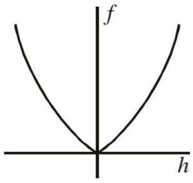
29. An ensemble of non-interacting spin-$\frac{1}{2}$ particles is in contact with a heat bath at temperature T and is subjected to an external magnetic field. Each particle can be in one of the two quantum states of energies $\pm\epsilon_0$. If the mean energy per particle is $-\epsilon_0/2$, then the free energy per particle is **[CSIR Dec. 2015]**

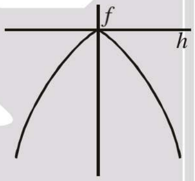
(a) $-2\epsilon_0\frac{\ln(4/\sqrt{3})}{\ln 3}$ (b) $-\epsilon_0\ln(3/2)$ (c) $-2\epsilon_0\ln 2$ (d) $-\epsilon_0\frac{\ln 2}{\ln 3}$

30. The partition function of a system of N Ising spins is $Z=\lambda_1^N+\lambda_2^N$, where λ_1 and λ_2 are functions of temperature, but are independent of N. If $\lambda_1>\lambda_2$, the free energy per spin in the limit $N\to\infty$ is **[NET Dec. 2015]**

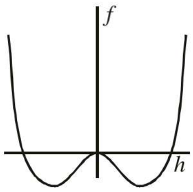
(a) $-k_BT\ln\left(\frac{\lambda_1}{\lambda_2}\right)$ (b) $-k_BT\ln\lambda_2$ (c) $-k_BT\ln(\lambda_1\lambda_2)$ (d) $-k_BT\ln\lambda_1$

31. Which of the following graphs shows the qualitative dependence of the free energy $f(h,T)$ of a ferromagnet in an external magnetic field h, and at a fixed temperature $T<T_c$, where T_c is the critical temperature?

(a)


(b)


[NET Dec. 2015]

(c)


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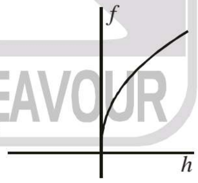


32. A box of volume V containing N molecules of an ideal gas, is divided by a wall with a hole into two compartments. If the volume of the smaller compartment is $V/3$, the variance of the number of particles in it, is

(a) $\frac{N}{3}$ (b) $\frac{2N}{9}$ (c) $\sqrt{N}$ (d) $\frac{\sqrt{N}}{3}$ **[NET June 2016]**

33. A gas of non-relativistic classical particles in one-dimension is subjected to a potential $V(x)=\alpha|x|$, where α is a constant). The partition function is $\left(\beta=\frac{1}{k_BT}\right)$ **[NET June 2016]**

(a) $\sqrt{\frac{4m\pi}{\beta^3\alpha^2h^2}}$ (b) $\sqrt{\frac{2m\pi}{\beta^3\alpha^2h^2}}$ (c) $\sqrt{\frac{8m\pi}{\beta^3\alpha^2h^2}}$ (d) $\sqrt{\frac{3m\pi}{\beta^3\alpha^2h^2}}$

34. Consider a gas of Cs atoms at a number density of 10^{12} atoms/cc. When the typical inter-particle distance is equal to the thermal de-Broglie wavelength of the particles, the temperature of the gas is nearest to (Take the mass of a Cs atom to be 22.7×10^{-26} kg). **[NET June 2016]**

(a) $1\times10^{-9}\,\text{K}$ (b) $7\times10^{-5}\,\text{K}$ (c) $1\times10^{-3}\,\text{K}$ (d) $2\times10^{-8}\,\text{K}$

35. Consider a gas of N classical particles in a two-dimensional square box of side L. If the total energy of the gas is E, the entropy (apart from an additive constant) is **[NET Dec. 2016]**

(a) $Nk_B\ln\left(\frac{L^2E}{N}\right)$ (b) $Nk_B\ln\left(\frac{LE}{N}\right)$ (c) $2Nk_B\ln\left(\frac{L\sqrt{E}}{N}\right)$ (d) $L^2k_B\ln\left(\frac{E}{N}\right)$

36. Consider a continuous time random walk. If a step has taken place at time $t=0$, the probability that the next step takes place between t and $t+dt$ is given by $bt\,dt$, where b is a constant. What is the average time between successive steps ? **[NET Dec. 2016]**

(a) $\sqrt{\frac{2\pi}{b}}$ (b) $\sqrt{\frac{\pi}{b}}$ (c) $\frac{1}{2}\sqrt{\frac{\pi}{b}}$ (d) $\sqrt{\frac{\pi}{2b}}$

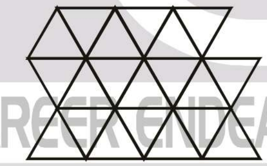
37. The partition function of a two-level system governed by the Hamiltonian $H=\begin{bmatrix}\gamma & -\delta\\ -\delta & -\gamma\end{bmatrix}$ is

(a) $2\sinh\left(\beta\sqrt{\gamma^2+\delta^2}\right)$ (b) $2\cosh\left(\beta\sqrt{\gamma^2+\delta^2}\right)$ **[NET Dec. 2016]**

(c) $\frac{1}{2}\left[\cosh\left(\beta\sqrt{\gamma^2+\delta^2}\right)+\sinh\left(\beta\sqrt{\gamma^2+\delta^2}\right)\right]$

(d) $\frac{1}{2}\left[\cosh\left(\beta\sqrt{\gamma^2+\delta^2}\right)-\sinh\left(\beta\sqrt{\gamma^2+\delta^2}\right)\right]$

38. Consider a random walk on an infinite two-dimensional triangular lattice, a part of which is shown in the figure below. **[NET Dec. 2016]**



If the probabilities of moving to any of the nearest neighbour sites are equal, what is the probability that the walker returns to the starting position at the end of exactly three steps ? **[NET Dec. 2016]**

(a) $\frac{1}{36}$ (b) $\frac{1}{216}$ (c) $\frac{1}{18}$ (d) $\frac{1}{12}$

39. An atom has a non-degenerate ground state and a doubly-degenerate excited state. The energy difference between the two states is ε. The specific heat at very low temperatures $(\beta\varepsilon \gg 1)$ is given by

(a) $k_B(\beta\varepsilon)$ (b) $k_B e^{-\beta\varepsilon}$ (c) $2k_B(\beta\varepsilon)^2e^{-\beta\varepsilon}$ (d) k_B **[NET Dec. 2016]**

40. The electrons in graphene can be thought of as a two-dimensional gas with a linear energy-momentum relation $E=|\vec{p}|v$, where $\vec{p}=(p_x, p_y)$ and v is a constant. If ρ is the number of electrons per unit area, the energy per unit area is proportional to **[NET Dec. 2016]**

(a) $\rho^{3/2}$ (b) ρ (c) $\rho^{1/3}$ (d) ρ^2

41. The single particle energy levels of a non-interacting three-dimensional isotropic system, labelled by momentum k, are proportional to k^3. The ratio $\vec{P}/\varepsilon$ of the average pressure $\vec{P}$ to the energy density ε at a fixed temperature, is **[NET June 2017]**

(a) 1/3 (b) 2/3 (c) 1 (d) 3

42. The Hamiltonian for three Ising spins S_0, S_1 and S_2, taking values ± 1, is $H = -jS_0(S_1 + S_2)$. If the system is in equilibrium at temperature T, the average energy of the system, in terms of $\beta = (k_BT)^{-1}$, is

(a) $-\dfrac{1+\cosh(2\beta j)}{2\beta\sinh(2\beta j)}$ (b) $-2j[1+\cosh(2\beta j)]$ **[NET June 2017]**

(c) $-2/\beta$ (d) $-2j\dfrac{\sinh(2\beta j)}{1+\cosh(2\beta j)}$

43. The dispersion relation of a gas of spin-$\frac{1}{2}$ fermions in two dimensions is $E = \hbar v|\vec{k}|$, where E is the energy, $\vec{k}$ is the wave vector and v is a constant with the dimension of velocity. If the Fermi energy at zero temperature is ε_F, the number of particles per unit area is **[NET Dec. 2017]**

(a) $\dfrac{\varepsilon_F}{(4\pi v\hbar)}$ (b) $\dfrac{\varepsilon_F^3}{(6\pi^2v^3\hbar^2)}$ (c) $\dfrac{\pi\varepsilon_F^{3/2}}{(3v^3\hbar^3)}$ (d) $\dfrac{\varepsilon_F^2}{(2\pi v^2\hbar^2)}$

44. The number of microstates of a gas of N particles in a volume V and of internal energy U, is given by

$$\Omega(U, V, N) = (V - Nb)^N\left(\frac{aU}{N}\right)^{3N/2},$$

(where a and b are positive constants). Its pressure P, volume V and temperature T, are related by

(a) $\left(P + \dfrac{aN}{V}\right)(V - Nb) = Nk_BT$ (b) $\left(P - \dfrac{aN}{V^2}\right)(V - Nb) = Nk_BT$ **[NET Dec. 2017]**

(c) $PV = Nk_BT$ (d) $P(V - Nb) = Nk_BT$

45. Consider a system of identical atoms in equilibrium with blackbody radiation in a cavity at temperature T. The equilibrium probabilities for each atom being in the ground state $|0\rangle$ and an excited state $|1\rangle$ are P_0 and P_1, respectively. Let n be the average number of photons in a mode in the cavity that causes transition between the two states. Let $W_{0\to 1}$ and $W_{1\to 0}$ denote, respectively, the squares of the matrix elements corresponding to the atomic transitions $|0\rangle \to |1\rangle$ and $|1\rangle \to |0\rangle$. Which of the following equations hold in equilibrium ?

(a) $P_0 nW_{0\to 1} = P_1W_{1\to 0}$ (b) $P_0W_{0\to 1} = P_1nW_{1\to 0}$ **[NET Dec. 2017]**

(c) $P_0nW_{0\to 1} = P_1W_{1\to 0} - P_1nW_{1\to 0}$ (d) $P_0nW_{0\to 1} = P_1W_{1\to 0} + P_1nW_{1\to 0}$

46. Two non-degenerate energy levels with energies 0 and ε are occupied by N non-interacting particles at a temperatures T. Using classical statistics, the average internal energy of the system is

(a) $\dfrac{N\varepsilon}{\left(1+e^{\varepsilon/k_BT}\right)}$ (b) $\dfrac{N\varepsilon}{\left(1-e^{\varepsilon/k_BT}\right)}$ (c) $N\varepsilon e^{-\varepsilon/k_BT}$ (d) $\dfrac{3}{2}Nk_BT$ **[NET Dec. 2017]**

47. Consider a particle diffusing in a liquid contained in a large box. The diffusion constant of the particle in the liquid is 1.0×10^{-2} cm^2/s. The minimum time after which the root-mean-squared displacement becomes more than 6 cm is

(a) 10 min (b) 6 min (c) 30 min (d) $\sqrt{6}$ min **[NET June 2018]**

48. The maximum intensity of solar radiation is at the wavelength of $\lambda_{\text{sun}} \sim 5000$ Å and corresponds to its surface temperature $T_{\text{sun}} \sim 10^4$ K. If the wavelength of the maximum intensity of an X-ray star is 5 Å, its surface temperature is of the order of **[NET June 2018]**

(a) 10^{16} K (b) 10^{14} K (c) 10^{10} K (d) 10^{7} K

49. The rotational energy levels of a molecule are $E_l = \frac{\hbar^2}{2I_0} l(l+1)$, where $l = 0, 1, 2, ...$ and I_0 is its moment of inertia. The contribution of the rotational motion to the Helmholtz free energy per molecule, at low temperature in a dilute gas of these molecules, is approximately **[NET Dec. 2018]**

(a) $-k_BT\left(1+\frac{\hbar^2}{I_0k_BT}\right)$ (b) $-k_BT\,e^{-\frac{\hbar^2}{I_0k_BT}}$ (c) $-k_BT$ (d) $-3k_BT\,e^{-\frac{\hbar^2}{I_0k_BT}}$

50. The vibrational motion of a diatomic molecule may be considered to be that of a simple harmonic oscillator with angular frequency ω. If a gas of these molecules is at a temperature T, what is the probability that a randomly picked molecule will be found in its lowest vibrational state ? **[NET Dec. 2018]**

(a) $1-e^{-\frac{\hbar\omega}{k_BT}}$ (b) $e^{-\frac{\hbar\omega}{2k_BT}}$ (c) $\tanh\left(\frac{\hbar\omega}{k_BT}\right)$ (d) $\frac{1}{2}\operatorname{cosech}\left(\frac{\hbar\omega}{2k_BT}\right)$

51. Consider an ideal Fermi gas in a grand canonical ensemble at a constant chemical potential. The variance of the occupation number of the single particle energy level with mean occupation number $\bar{n}$ is

(a) $\bar{n}(1-\bar{n})$ (b) $\sqrt{\bar{n}}$ (c) $\bar{n}$ (d) $1/\sqrt{\bar{n}}$ **[NET Dec. 2018]**

52. The Hamiltonian of a one-dimensional Ising model of N spins (N large) is

$$H = -J\sum_{i=1}^{N}\sigma_i\sigma_{i+1}$$

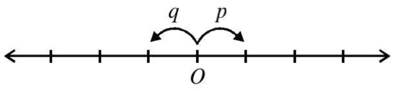
where the spin $\sigma_i = \pm 1$ and J is a positive constant. At inverse temperature $\beta = \frac{1}{k_BT}$, the correlation function between the nearest neighbour spins $\langle\sigma_i\sigma_{i+1}\rangle$ is **[NET Dec. 2018]**

(a) $\frac{e^{-\beta J}}{\left(e^{\beta J}+e^{-\beta J}\right)}$ (b) $e^{-2\beta J}$ (c) $\tanh(\beta J)$ (d) $\coth(\beta J)$

53. At low temperatures, in the Debye approximation, the contribution of the phonons to the heat capacity of a two-dimensional solid is proportional to **[NET Dec. 2018]**

(a) T^2 (b) T^3 (c) $T^{1/2}$ (d) $T^{3/2}$

54. A particle hops on a one-dimensional lattice with lattice spacing a. The probability of the particle to hop to the neighbouring site to its right is p, while the corresponding probability to hop to the left is $q = 1-p$. The root-mean-squared deviation $\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$ in displacement after N steps, is **[NET Dec. 2018]**



(a) $a\sqrt{Npq}$ (b) $aN\sqrt{pq}$ (c) $2a\sqrt{Npq}$ (d) $a\sqrt{N}$

55. The energy levels accessible to a molecule have energies $E_1 = 0,\ E_2 = \Delta$ and $E_3 = 2\Delta$ (where Δ is a constant). A gas of these molecules is in thermal equilibrium at temperature T. The specific heat at constant volume in the high temperature limit $(k_B T \gg \Delta)$ varies with temperature as **[NET Dec. 2018]**

(a) $1/T^{3/2}$ (b) $1/T^3$ (c) $1/T$ (d) $1/T^2$

56. At each time step, a random walker in one-dimension either remains at the same point with probability $\frac{1}{4}$, or moves by a distance Δ to the right or left with probabilities 3/8 each. After N time steps, its root mean squared displacement is **[NET June 2019]**

(a) $\Delta\sqrt{N}$ (b) $\Delta\sqrt{\frac{9N}{16}}$ (c) $\Delta\sqrt{\frac{3N}{4}}$ (d) $\Delta\sqrt{\frac{3N}{8}}$

57. The Hamiltonian of three Ising spins S_1, S_2 and S_3, each taking values ±1, is $H = -J(S_1S_2 + S_2S_3) - hS_1$, where J and h are positive constants. The mean value of S_3 in equilibrium at a temperatrue $T = 1/(k_B\beta)$, is

(a) $\tanh^3(\beta J)$ (b) $\tanh(\beta h)\tanh^2(\beta J)$ **[NET June 2019]**

(c) $\sinh(\beta h)\sinh^2(\beta J)$ (d) 0

58. The free energy of a magnetic system, as a function of its magnetisation m, is $F = \frac{1}{2}am^2 - \frac{1}{4}bm^4 + \frac{1}{6}m^6$, where a and b are positive constants. At a fixed value of a, the critical value of b, above which the minimum of F will be at a non-zero value of magnetisation, is **[NET June 2019]**

(a) $\sqrt{10a/3}$ (b) $\sqrt{16a/3}$ (c) $\frac{10}{3}\sqrt{a}$ (d) $\frac{16}{3}\sqrt{a}$

59. The range of the inter-atomic potential in gaseous hydrogen is approximately 5Å. In thermal equilibrium, the maximum temperature for which the atom-atom scattering is dominantly s-wave, is

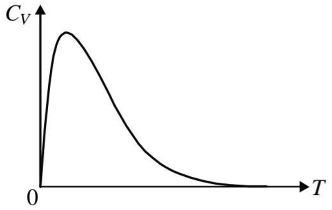
(a) 500 K (b) 100 K (c) 1 K (d) 1 mK **[NET June 2019]**

60. The angular frequency of oscillation of a quantum harmonic oscillator in two dimensions is ω. It it is contact with an external heat bath at temperature T, its partition function is (in the following $\beta = \frac{1}{k_B T}$)

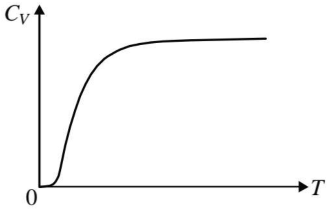
(a) $\frac{e^{2\beta\hbar\omega}}{\left(e^{2\beta\hbar\omega}-1\right)^2}$ (b) $\frac{e^{\beta\hbar\omega}}{\left(e^{\beta\hbar\omega}-1\right)^2}$ (c) $\frac{e^{\beta\hbar\omega}}{e^{\beta\hbar\omega}-1}$ (d) $\frac{e^{2\beta\hbar\omega}}{e^{2\beta\hbar\omega}-1}$ **[NET Dec. 2019]**

61. The energies available to a three-state system are 0, E and $2E$, where $E > 0$. Which of the following graphs best represents the temperature dependence of the specific heat ? **[NET Dec. 2019]**

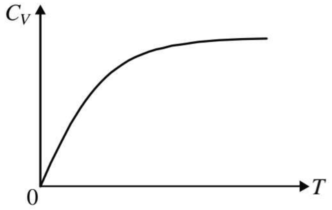
(a)



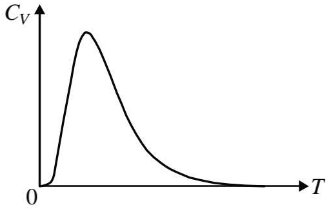
(b)



(c)



(d)



62. The Hamiltonian of two particles, each of mass m, is **[NET Dec. 2019]**

$$H(q_1, p_1; q_2, p_2) = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + k\left(q_1^2 + q_2^2 + \frac{1}{4}q_1 q_2\right), \text{ where } k > 0 \text{ is a constant.}$$

The value of the partition function $Z(\beta) = \int_{-\infty}^{\infty} dq_1 \int_{-\infty}^{\infty} dp_1 \int_{-\infty}^{\infty} dq_2 \int_{-\infty}^{\infty} dp_2\, e^{-\beta H(q_1, p_1; q_2, p_2)}$ is

(a) $\frac{2m\pi^2}{k\beta^2}\sqrt{\frac{16}{15}}$ (b) $\frac{2m\pi^2}{k\beta^2}\sqrt{\frac{15}{16}}$ (c) $\frac{2m\pi^2}{k\beta^2}\sqrt{\frac{63}{64}}$ (d) $\frac{2m\pi^2}{k\beta^2}\sqrt{\frac{64}{63}}$

63. A particle hops randomly from a site to its nearest neighbour in each step on a square lattice of unit lattice constant. The probability of hopping to the positive x-direction is 0.3, to the negative x-direction is 0.2, to the positive y-direction is 0.2 and to the negative y-direction is 0.3. If a particle starts from the origin, its mean position after N steps is **[NET Dec. 2019]**

(a) $\frac{1}{10}N(-\hat{i} + \hat{j})$ (b) $\frac{1}{10}N(\hat{i} - \hat{j})$ (c) $N(0.3\hat{i} - 0.2\hat{j})$ (d) $N(0.2\hat{i} - 0.3\hat{j})$

64. The total internal energy of a system of N distinguishable particles (where N is large) is U. Each particle may be in the ground state of energy 0, or in an excited state of energy $\in$. The entropy of the system is

(a) $k_B\left[N \ln N + \left(N - \frac{U}{\in}\right)\ln\left(N - \frac{U}{\in}\right) + \frac{U}{\in}\ln\frac{U}{\in}\right]$ **[NET Dec. 2019]**

(b) $k_B\left[N \ln N - \left(N - \frac{U}{\in}\right)\ln\left(N - \frac{U}{\in}\right) + \frac{U}{\in}\ln\frac{U}{\in}\right]$

(c) $k_B\left[N \ln N - \left(N - \frac{U}{\in}\right)\ln\left(N - \frac{U}{\in}\right) - \frac{U}{\in}\ln\frac{U}{\in}\right]$

(d) $k_B \ln N$

65. The number density of mono-atomic gas in 10^{18} m^{-3}. The temperature at which the thermal de Broglie wavelength of the atoms (which have mass m) equals the average inter-atomic separation, is **[NET Dec. 2019]**

(a) $\frac{\hbar^2}{2\pi m k_B}\times 10^{12}K$ (b) $\frac{\hbar^2}{2\pi m k_B}\times 10^{10}K$ (c) $\frac{\hbar^2}{2\pi m k_B}\times 10^{9}K$ (d) $\frac{\hbar^2}{2\pi m k_B}\times 10^{11}K$

66. Two spheres are subjected to black-body radiation at temperature T_1 and T_2 respectively. For the first sphere only half of its surface is exposed to the radiation, while the full surface of the second sphere is exposed. The ratio of the radii of the spheres are $\frac{R_1}{R_2} = 2$. If the forces due to radiation pressure on the two spheres are the same, the ratio T_1/T_2 is **[NET Dec. 2019]**

(a) $2^{1/4}$ (b) $1/\sqrt{2}$ (c) 1 (d) $1/2^{1/4}$

67. A material of inhomogeneous thermal conductivity $\kappa(\vec{r})$ has a uniform density ρ and heat capacity C_V. The heat equation obeyed by the time-dependent temperature distribution $T(\vec{r},t)$, is **[NET Dec. 2019]**

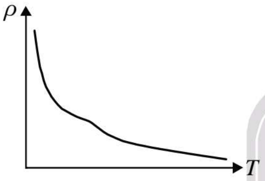
(a) $\rho C_V \dfrac{\partial T(\vec{r},t)}{\partial t} = \vec{\nabla}\cdot\left(\kappa(\vec{r})\vec{\nabla}T(\vec{r},t)\right)$ (b) $\rho C_V \dfrac{\partial T(\vec{r},t)}{\partial t} = \kappa(\vec{r})\nabla^2 T(\vec{r},t)$

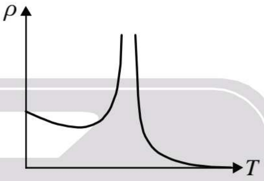
(c) $\rho C_V \dfrac{\partial T(\vec{r},t)}{\partial t} = \nabla^2\left(\kappa(\vec{r})T(\vec{r},t)\right)$ (d) $\rho C_V \dfrac{\partial T(\vec{r},t)}{\partial t} = \left(\vec{\nabla}\kappa(\vec{r})\right)\cdot\left(\vec{\nabla}T(\vec{r},t)\right)$

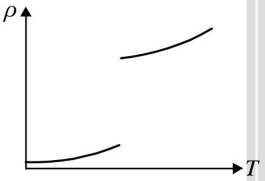
68. The Hamiltonian of a system of three Ising spins is $H = -J(S_1S_2 + S_2S_3)$, where each spin $S_i = \pm 1$ and $J > 0$. Then at thermal equilibrium at temperature T, the correlation function $\langle S_1 S_2\rangle$ is **[NET Dec. 2019]**

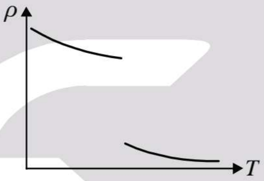
(a) 0 (b) $\text{sech}\,\dfrac{J}{k_BT}$ (c) $\tanh\dfrac{J}{k_BT}$ (d) $\left(\tanh\dfrac{J}{k_BT}\right)^2$

69. Consider raising the temperature of water from 70 °C to 130 °C at one atmospheric pressure. Which of the following plots best represents the dependence of density ρ on the temperature T? **[NET Dec. 2019]**

(a)


(b)


(c)


(d)


ANSWER KEY

1. (b) 2. (a) 3. (a) 4. (c) 5. (d) 6. (a) 7. (d)
8. (b) 9. (d) 10. (c) 11. (c) 12. (b) 13. (b) 14. (a)
15. (c) 16. (d) 17. (c) 18. (c) 19. (c) 20. (d) 21. (a)
22. (b) 23. (c) 24. (a) 25. (d) 26. (a) 27. (c) 28. (d)
29. (a) 30. (d) 31. (b) 32. (b) 33. (c) 34. (d) 35. (a)
36. (d) 37. (b) 38. (c) 39. (c) 40. (a) 41. (c) 42. (d)
43. (d) 44. (d) 45. (d) 46. (a) 47. (a) 48. (d) 49. (d)
50. (a) 51. (a) 52. (c) 53. (a) 54. (c) 55. (d) 56. (c)
57. (c) 58. (b) 59. (c) 60. (b) 61. (d) 62. (d) 63. (b)
64. (c) 65. (a) 66. (d) 67. (a) 68. (c) 69. (d)

JEST Previous Years' Questions

1. Consider a system of particles in three dimensions with momentum $\vec{p}$ and energy $E = c.\vec{P}|$, c being a constant. The system is maintained at inverse temperature β. volume V and chemical μ. What is the grand particle partition function of the system ? **[JEST 2012]**

(a) $\exp\left[e^{\beta\mu}8\pi V/(\beta ch)^3\right]$ (b) $e^{\beta\mu}6\pi V/(\beta ch)^2$ (c) $\exp\left[e^{\beta\mu}6\pi V/(\beta ch)^3\right]$ (d) $e^{\beta\mu}8\pi V/(\beta ch)^2$

2. Consider a system maintained at temperature T, With two available energy states E_1 and E_2 each with degeneracies g_1 and g_2. If p_1 and p_2 are probabilities of occupancy of the energy states, what is the entropy of the system ? **[JEST 2012]**

(a) $S = -\kappa_B\left[p_1 \ell n(p_1/g_1) + p_2 \ell n(p_2/g_2)\right]$ (b) $S = -\kappa_B\left[p_1 \ell n(p_1 g_1) + p_2 \ell n(p_2 g_2)\right]$

(c) $S = -\kappa_B\left[p_1 \ell n\left(p_1^{g_1}\right) + p_2 \ell n\left(p_2^{g_2}\right)\right]$ (d) $S = -\kappa_B\left[(1/p_1)\ell n(p_1/g_1) + (1/p_2)\ell n(p_2/g_2)\right]$

3. Consider a system of two particles A and B. Each particle can occupy one of three possible quantum states $|1\rangle, |2\rangle$, and $|3\rangle$. The ratio of the probability that the two particles are in the same state to the probability that the two particles are in different states is calculated for bosons and classical (Maxwll-Boltzman) particles. They are respectively **[JEST 2013]**

(a) 1,0 (b) 1/2,1 (c) 1,1/2 (d) 0,1/2

4. A monoatomic gas consists of atoms with two internal energy levels, ground state $E_0 = 0$ an an excited state $E_1 = E$. The specific heat of the gas is given by **[JEST 2014]**

(a) $\frac{3}{2}k$ (b) $\frac{E^2 e^{E/kT}}{kT^2\left(1+e^{E/kT}\right)^2}$

(c) $\frac{3}{2}k + \frac{E^2 e^{E/kT}}{kT^2\left(1+e^{E/kT}\right)^2}$ (d) $\frac{3}{2}k - \frac{E^2 e^{E/kT}}{kT^2\left(1+e^{E/kT}\right)^2}$

5. For non-interacting Fermions in *d*-dimensions, the density of states $D(E)$ varies as $E^{(d/2-1)}$. The Fermi energy E_F of an *N* particle system in 3-, 2- and 1-dimensions will scale respectively as, **[JEST 2015]**

(a) $N^2, N^{2/3}, N$ (b) $N, N^{2/3}, N^2$ (c) $N, N^2, N^{2/3}$ (d) $N^{2/3}, N, N^2$

6. A particle in thermal equilibrium has only 3 possible states with energies $-\varepsilon, 0, \varepsilon$. If the system is maintained at a temperature $T \gg \frac{\varepsilon}{k_B}$, then the average energy of the particle can be approximated to, **[JEST 2015]**

(a) $\frac{2\varepsilon^2}{3k_BT}$ (b) $-\frac{2\varepsilon^2}{3k_BT}$ (c) $-\frac{\varepsilon^2}{k_BT}$ (d) 0

7. Consider *N* non-interacting electrons $(N \sim N_A)$ in a box of sides L_x, L_y, L_z. Assume that the dispersion relation is $\varepsilon(k) = Ck^4$, where *C* is a constant, the ratio of the ground state energy per particle to the Fermi energy is : **[JEST 2016]**

(a) $\frac{3}{7}$ (b) $\frac{7}{3}$ (c) $\frac{3}{5}$ (d) $\frac{5}{7}$

8. A gas of N molecules of mass m is confined in a cube of volume $V = L^3$ at temperature T. The box is in a uniform gravitational field $-g\hat{z}$. Assume that the potential energy of a molecule is $U = mgz$, where $z \in [0, L]$ is the vertical coordinate inside the box. The pressure $P(z)$ at height z is :

(a) $P(z) = \dfrac{N}{V}\dfrac{mgL}{2}\dfrac{\exp\left(-\dfrac{mg(z-L/2)}{k_BT}\right)}{\sinh\left(\dfrac{mgL}{2k_BT}\right)}$ (b) $P(z) = \dfrac{N}{V}\dfrac{mgL}{2}\dfrac{\exp\left(-\dfrac{mg(z-L/2)}{k_BT}\right)}{\cosh\left(\dfrac{mgL}{2k_BT}\right)}$ **[JEST 2016]**

(c) $P(z) = \dfrac{k_BTN}{V}$ (d) $P(z) = \dfrac{N}{V}mgz$

9. A two dimensional box in a uniform magnetic field B contains $N/2$ localised spin-$\frac{1}{2}$ particles with magnetic moment μ, and $N/2$ free spinless particles which do not interact with each other. The average energy of the system at a temperature T is : **[JEST 2016]**

(a) $3NkT - \dfrac{1}{2}N\mu B \sinh(\mu B/k_BT)$ (b) $NkT - \dfrac{1}{2}N\mu B \tanh(\mu B/k_BT)$

(c) $\dfrac{1}{2}NkT - \dfrac{1}{2}N\mu B \tanh(\mu B/k_BT)$ (d) $\dfrac{3}{2}NkT + \dfrac{1}{2}N\mu B \cosh(\mu B/k_BT)$

10. Suppose that the number of microstates available to a system of N particles depends on N and the combined variable UV^2, where U is the internal energy and V is the volume of the system. The system initially has volume 2m^3 and energy 200 J. It undergoes an isentropic expansion to volume 4m^3. What is the final pressure of the system in SI units ? **[JEST 2017]**

11. A system of particles of N lattice sites is in equilibrium at temperature T and chemical potential μ. Multiple occupancy of the sites is forbidden. The binding energy of a particle at each site is $-\varepsilon$. The probability of on site being occupied is **[JEST 2017]**

(a) $\dfrac{1-e^{\beta(\mu+\varepsilon)}}{1-e^{(N+1)\beta(\mu+\varepsilon)}}$ (b) $\dfrac{1}{\left[1+e^{\beta(\mu+\varepsilon)}\right]^N}$ (c) $\dfrac{1}{\left[1+e^{-\beta(\mu+\varepsilon)}\right]^N}$ (d) $\dfrac{1-e^{-\beta(\mu+\varepsilon)}}{1-e^{-(N+1)\beta(\mu+\varepsilon)}}$

12. Two classical particles are distributed among $N\ (>2)$ sites on a ring. Each site can accommodate only one particle. If two particles occupy two nearest neighbour sites, then the energy of the system is increased by ε. The average energy of the system at temperature T is **[JEST 2017]**

(a) $\dfrac{2\varepsilon e^{-\beta\varepsilon}}{(N-3)+2e^{-\beta\varepsilon}}$ (c) $\dfrac{2N\varepsilon e^{-\beta\varepsilon}}{(N-3)+2e^{-\beta\varepsilon}}$ (c) $\dfrac{\varepsilon}{N}$ (d) $\dfrac{2\varepsilon e^{-\beta\varepsilon}}{(N-2)+2e^{-\beta\varepsilon}}$

13. A cylinder at temperature $T = 0$ is separated into two compartments A and B by a free sliding piston. Compartments A and B are filled by Fermi gases made of spin 1/2 and 3/2 particles respectively, If particles in both the compartments have same mass, the ratio of equilibrium density of the gas in compartment A to that of gas in compartment B is **[JEST 2017]**

(a) 1 (b) $\dfrac{1}{3^{2/5}}$ (c) $\dfrac{1}{2^{2/5}}$ (d) $\dfrac{1}{2^{2/3}}$

14. When a collection of two-level systems is in equilibrium at temperature T_0, the ratio of the population in the lower and upper levels is 2 : 1. When the temperature is changed to T, the ratio is 8 : 1. Then

(a) $T = 2T_0$ (b) $T_0 = 2T$ (c) $T_0 = 3T$ (d) $T_0 = 4T$ **[JEST 2018]**

15. A collection of N interacting magnetic moments, each of magnitude μ, is subjected to a magnetic field H along the z-direction. Each magnetic moment has a doubly degenerate level of energy zero, and two non-degenerate levels of energies –μH and μH respectively. The collection is in thermal equilibrium at temperature T. The total energy $E(T, H)$ of the collection is **[JEST 2018]**

(a) $-\dfrac{\mu HN\sinh\left(\mu H/k_BT\right)}{1+\cosh\left(\mu H/k_bT\right)}$ (b) $-\dfrac{\mu HN}{2\left(1+\cosh\left(\mu H/k_BT\right)\right)}$

(c) $-\dfrac{\mu HN\cosh\left(\mu H/k_BT\right)}{1+\cosh\left(\mu H/k_bT\right)}$ (d) $-\mu HN\dfrac{\sinh\left(\mu H/k_BT\right)}{\cosh\left(\mu H/k_BT\right)}$

16. Consider a grand canonical ensemble of a system of one dimensional non-interacting classical harmonic oscillators (each of frequency ω). Which one of the following equations is correct ? Here the angular bracket $\langle\bullet\rangle$ indicate the ensemble average. N, E and T represent the number of particles, energy and temperature, respectively. k_B is the Boltzmann constant. **[JEST 2019]**

(a) $\langle E\rangle = N\dfrac{k_BT}{2}$ (b) $\langle E\rangle = \langle N\rangle\dfrac{k_BT}{2}$ (c) $\langle E\rangle = Nk_BT$ (d) $\langle E\rangle = \langle N\rangle k_BT$

ANSWER KEY

1. (a)
2. (a)
3. (c)
4. (c)
5. (d)
6. (b)
7. (a)
8. (a)
9. (c)
10. (0025)
11. (b)
12. (a)
13. (c)
14. (c)
15. (a)
16. (d)

TIFR Previous Years' Questions

1. A closed, thermally-insulated box contains one mole of an ideal monatomic gas G in thermodynamic equilibrium with blackbody radiation B. The total internal energy of the system is $U = U_G + U_B$ where U_G and $U_B(\alpha T^4)$ are the energies of the ideal gas and the radiation respectively. If $U_G = U_B$ at a certain temperature T_0 K, then the energy required to raise the temperature from T_0 K to $(T_0 + 1)$ K, in terms of the gas constant R, is

(a) 7.5 R (b) 6 R (c) 1.5 R (d) 0.33 R **[TIFR 2011]**

2. Consider two energies of a free electron gas in a metal at an absolute temperature T, viz., $E_{\pm} = E_F \pm \Delta$ where E_F is the Fermi level. If the corresponding electron populations $n(E_{\pm})$ satisfy the relation $n(E_-)/n(E_+) = 2$, then $\Delta =$ **[TIFR 2013]**

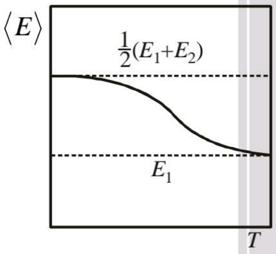
(a) $k_B T \ln 2$ (b) $2k_B T$ (c) $\frac{k_B T}{2}$ (d) $k_B T$

3. A binary star is observed to consist of a blue star B (peak wavelength 400 nm) and a red star R (peak wavelength 800 nm) orbiting each other. As observed from the Earth, B and R appear equally bright. Assuming that the stars radiate as perfect blackbodies, it follows that the ratio of volumes V_B/V_R of the two stars is **[TIFR 2013]**

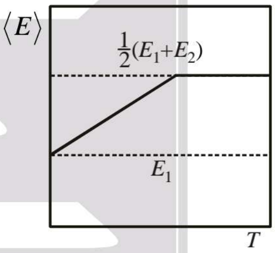
(a) $\frac{1}{64}$ (b) 64 (c) 16 (d) $\frac{1}{16}$

4. Consider an ensemble of microscopic quantum mechanical systems with two energy levels E_1 and E_2, where $E_1 < E_2$. Which of the following graphs best describes the temperature dependence of the average energy $\langle E \rangle$ of the system? **[TIFR 2013]**

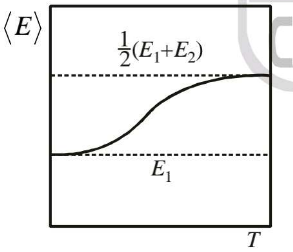
(a)



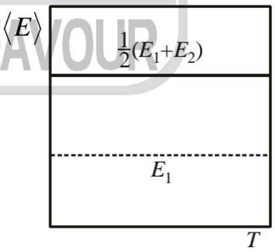
(b)



(c)

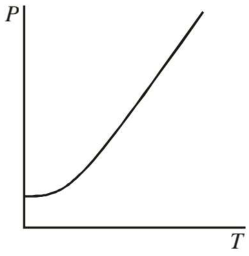


(d)

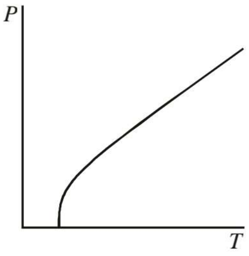


5. Which of the following graphs qualitatively describes the pressure P of a gas of non-interacting fermions in thermal equilibrium at a constant volume as a function of temperature ? **[TIFR 2015]**

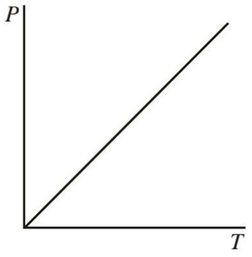
(a)



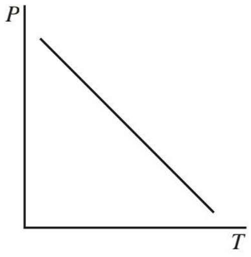
(b)



(c)



(d)



6. In a monatomic gas, the first excited state is only 1.5 eV above the ground state, while the other excited states are much higher up. The ground state is doubly-degenerate, while the first excited state has a four-fold degeneracy. If now, the gas is heated to a temperature of 7000 K, the fraction of atoms in the excited state will be approximately **[TIFR 2015]**
(a) 0.14 (b) 0.07 (c) 0.42 (d) 0.3

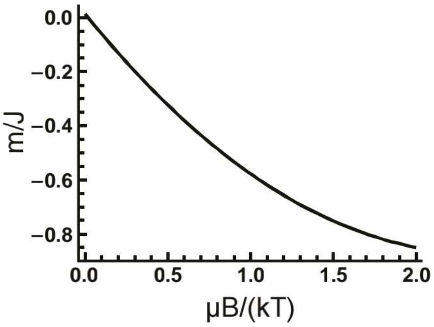
7. The energy per oscillator of an isolated system of a large number of identical non-interacting fermions in a one-dimensional harmonic oscillator potential is $5\hbar\omega/4$, where ω is the angular frequency of the harmonic oscillator. The entropy of the system per oscillator is given by **[TIFR 2016]**
(a) 0.25 (b) 0.56 (c) 0.63 (d) 0.75

8. N distinguishable particles are distributed among three states having energies $E = 0, k_BT$ and $2k_BT$ respectively. If the total equilibrium energy of the system is $138.06\,k_BT$, find the number N of particles. **[TIFR 2016]**

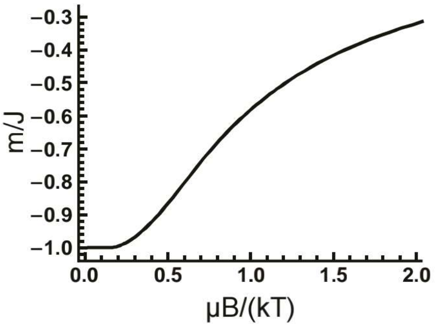
9. A gas of non-interacting particles, each of rest mass 1 MeV, is at a temperature $T = 2.0 \times 10^7$ K and has an average particle density $n = 2.7 \times 10^{34}\ \text{cm}^{-3}$. We can obtain a reasonably correct treatment of this system
(a) only by using special relativity as well as quantum mechanics **[TIFR 2016]**
(b) by neglecting quantum mechanics but not special theory of relativity
(c) by neglecting special relativity but not quantum mechanics
(d) by neglecting both special relativity and quantum mechanics

10. Consider a system of non-interacting particles with integer angular momentum J at a temperature T. This system is placed in a magnetic field B in the z-direction. The energy of a state with $J_z = m\hbar$ is $E_m = m\mu_B B$ with $\mu_B > 0$. The fractional magnetization of the particles as a function of $\mu_B B/k_BT$ can be represented as **[TIFR 2017]**

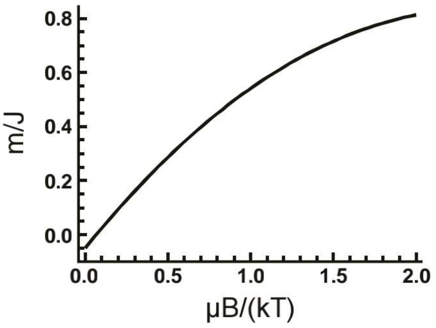
(a)



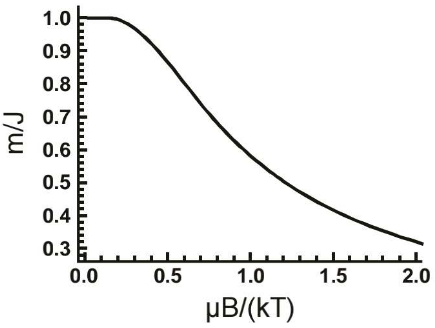
(b)



(c)



(d)



11. A system of particles occupying single-particle levels and obeying Maxwell-Boltzmann statistics is in thermal equilibrium with a heat reservoir at temperature T. If the population distribution in the non-degenerate energy levels is as shown in the table below, what would be the temperature of the system in degree Kelvin ? **[TIFR 2017]**

Energy (eV)	Population %
30.30	3.16
21.60	8.69
13.01	23.54
4.31	64.61

12. Two identical bosons may occupy any of two energy levels $0,\ \varepsilon$, where $\varepsilon > 0$. The lowest energy state is doubly-degenerate and the excited state is non-degenerate. Assume that the two-particle system is in thermal equilibrium at a temperature T. Calculate the average energy $\langle E\rangle$. What will be the leading term of

$$\frac{\langle E\rangle}{\exp\left(-\frac{\varepsilon}{k_B T}\right)}$$

at low temperature ? **[TIFR 2017]**

13. The cosmic microwave background radiation in the Universe has a blackbody distribution corresponding to a temperature 2.735 K. In a certain cosmological model, it was assumed that the universe consists purely of radiation and is undergoing adiabatic expansion. In this model it was predicted that the volume of the Universe will be tripled in the next 10^{10} years. The corresponding blackbody radiation temperature would be
(a) 0.9116 K (b) 2.078 K (c) 1.896 K (d) 1.526 K **[TIFR 2017]**

14. N-particles are distributed among three energy levels having energies: $0,\ k_BT$ and $2k_BT$ respectively. If the total equilibrium energy of the system is approximately 42.5 k_BT then find the value of N (to the closest integer). **[TIFR 2018]**

15. A statistical system, kept at a temperature T, has n discrete energy levels with equal level spacing ε, starting from the energy 0. If, now, a single particle is placed in the system what will be the mean energy of the system in the limit as $n \to \infty$? [The answer should not be left as a summation]. **[TIFR 2018]**

16. In an experiment to measure the Earth's mean albedo (i.e., fraction of solar energy reflected back into space), the solar constant (i.e., flux of solar energy incident upon the Earth), was measured as 1.37 kW m^{-2}. Assuming the Earth to behave as a perfect blackbody at a uniform surface temperature of –18 °C, the albedo is about
(a) 0.30 (b) 0.18 (c) 0.06 (d) 0.46 **[TIFR 2019]**

17. An ideal monatomic gas at chemical potential $\mu = -1$ eV and a temperature given by $k_B T = 0.1$ eV is in equilibrium with an adsorbing metal surface, i.e., there are isolated sites distributed randomly on the metal surface where the gas atom can get bound. Each such binding site can adsorb 0, 1 or 2 atoms with the released energy being 0, –1 eV and –1.9 eV respectively. The average number of adsorbed molecules at each site would be **[TIFR 2019]**

(a) $\frac{2+e}{1+2e}$ (b) $\frac{1+2e}{1+e}$ (c) $\frac{1+e}{1+2e}$ (d) $\frac{1+e}{1-e}$

18. At low temperature, the measured specific heat C_V of a sample is found to depend on temperature as $C_V = aT^{3/2} + bT^3$, where a and b are constants. This material has **[TIFR 2019]**

(a) one fermionic excitation with dispersion relation $\omega \propto k^4$;
another bosonic excitation with dispersion relation $\omega \propto k$

(b) one fermionic excitation with dispersion relation $\omega \propto k^2$;
another fermionic excitation with dispersion relation $\omega \propto k$

(c) one fermionic excitation with dispersion relation $\omega \propto k^2$;
another bosonic excitation with dispersion relation $\omega \propto k^4$

(d) one bosonic excitation with dispersion relation $\omega \propto k^2$;
another bosonic excitation with dispersion relation $\omega \propto k$

19. Consider N non-interacting distinguishable particle in equilibrium at an absolute temperature T. Each particle can only occupy one of two possible states of energy 0 and ε respectively $(\varepsilon > 0)$. The entropy of the system, in terms of $\beta = \frac{\varepsilon}{k_B T}$ is **[TIFR 2019]**

(a) $Nk_B\left\{\ln(1+e^{-\beta}) - \frac{e^{-\beta}}{1+e^{-\beta}}\right\}$ (b) $Nk_B\left\{\ln(1+e^{-\beta}) - \frac{e^{-\beta}}{1-e^{-\beta}}\right\}$

(c) $Nk_B\left\{\ln(1-e^{-\beta}) + \frac{\beta e^{-\beta}}{1-e^{-\beta}}\right\}$ (d) $Nk_B\left\{\ln(1+e^{-\beta}) + \frac{\beta e^{-\beta}}{1+e^{-\beta}}\right\}$

ANSWER KEY

1. (a)	**2. (a)**	**3. (a)**	**4. (c)**	**5. (a)**	**6. (a)**	**7. (b)**
8. (325)	**9. (c)**	**10. (a)**	**11. ()**	**12.** $\left(\frac{2\varepsilon}{3}\right)$	**13. (c)**	**14. (100)**
15. (*)	**16. (a)**	**17. (a)**	**18. (d)**	**19. (d)**		

Other Examinations Previous Years' Questions

1. E_1 and E_2 is non degenerate energy then probability of E_2.

(a) $\dfrac{\exp(\beta E_1)}{\exp(\beta E_1)+\exp(\beta E_2)}$ (b) $\dfrac{\exp(\beta E_1)+\exp(\beta E_2)}{\exp(-\beta E_1)+\exp(-\beta E_2)}$

(c) $\dfrac{\exp(-\beta E_1)}{\exp(-\beta E_2)+\exp(-\beta E_1)}$ (d) $\dfrac{\exp(-\beta E_2)}{\exp(-\beta E_1)+\exp(-\beta E_2)}$

2. N molecules of an ideal gas are in a container of volume V_0. If a molecule has the same probability of being at any point in the container, the probability of M of them being in a smaller volume V in the container is:

(a) M/N (b) $\dfrac{V^M (V_0-V)^{N-M}}{V_0^N}$

(c) $\dfrac{N!}{(N-M)!M!}\dfrac{V^M (V_0-V)^{N-M}}{V_0^N}$ (d) $\dfrac{N!}{(N-M)!M!}\dfrac{V_0^M (V_0-V)^{N-M}}{V^N}$

3. A non-interacting system has two energy levels, 0 and ε. The lower level is doubly degenerate while that of energy ε is non-degenerate. If the system is in thermal equilibrium at temperature T, the single - particle partitin function is:

(a) $Z=2e^{-\varepsilon/k_BT}$ (b) $Z=1+2e^{-\varepsilon/k_BT}$ (c) $Z=2+e^{-\varepsilon/k_BT}$ (d) $Z=2\left(1+e^{-\varepsilon/k_BT}\right)$

4. Let $\sigma_U^2 \equiv \left\langle U-\langle U\rangle^2\right\rangle$ and $\sigma_P^2 = \left\langle P-\langle P\rangle^2\right\rangle$ be the variances of the internal energy U and the pressure P about their mean values $\langle U\rangle$ and $\langle P\rangle$ in a system of N particles at constant temperature. Then

(a) $\sigma_U^2 \propto N$ and $\sigma_P^2 \propto N$ (b) $\sigma_U^2 \propto \dfrac{1}{N}$ and $\sigma_P^2 \propto \dfrac{1}{N}$

(c) $\sigma_U^2 \propto \dfrac{1}{N}$ and $\sigma_P^2 \propto N$ (d) $\sigma_U^2 \propto N$ and $\sigma_P^2 \propto \dfrac{1}{N}$

5. A closed system can be in any one of two nondegenerate microstates with energy E_1 and E_2. The system is in thermal equilibrium with a heat bath at temperature T. The probability that the system will be observed in the state with energy E_2 is given by

(a) $\exp\left(-\dfrac{E_2}{k_BT}\right)$ (b) $\exp\left(-\dfrac{E_1-E_2}{k_BT}\right)$

(c) $\dfrac{\exp\left(-\dfrac{E_2}{k_BT}\right)}{\exp\left(-\dfrac{E_1}{k_BT}\right)+\exp\left(-\dfrac{E_2}{k_BT}\right)}$ (d) $\dfrac{\exp\left(-\dfrac{E_1-E_2}{k_BT}\right)}{\exp\left(-\dfrac{E_1}{k_BT}\right)+\exp\left(-\dfrac{E_2}{k_BT}\right)}$

6. If the volume of a three- dimensional free Fermi gas is decreased by a factor of 8 at zero temperature. its internal energy changes by a factor of

(a) 8 (b) 2 (c) 4 (d) $1/8$

7. An ideal Bose gas undergoes Bose-Einstein condensation at temperature T_0. At any finite temperature below T_0,

(a) the chemical potential is equal to zero (b) only the lowest energy level is occupied
(c) the internal energy is equal to zero (d) the pressure is equal to zero.

8. Two non-interacting bosons are distributed in three degenerate levels of energy zero. The canonical partition function of this system is

(a) 4.5 (b) 6 (c) 9 (d) 8

9. If the volume of a three-dimensional system of non-interacting fermions is decreased by a factor of 27 at zero temperature, its, internal energy change by a factor of

(a) 9 (b) 3 (c) 1/27 (d) 1/9

10. Two localized non-interacting spin 1/2 magnetic ions of magnetic moment μ are placed in an external magnetic field H, at temperature T. If $k_B T >> \mu H$, then the entropy of the system is, to a good approximation.

(a) $S = k_B \ln 2$ (b) $S = 2k_B \ln 2$ (c) $S = 3k_B \ln 2$ (d) $S = 4k_B \ln 2$

11. If P_1, P_2 and P_3 represent the pressures of ideal B.E. ; M.B. and F.D. gases at low temperature, respectively then

(a) $P_1 < P_2 ; P_1 < P_3$ (b) $P_1 < P_3 ; P_1 > P_2$ (c) $P_1 > P_2 ; P_1 > P_3$ (d) $P_1 = P_2 = P_3$

12. Consider a random walk of N steps, each of unit length but randomly and independently oriented with respect to each other in a two dimensional space. In the limit $N \to \infty$, the mean square distance of the end point from the starting point is proportional to

(a) N^2 (b) $\sqrt{N}$ (c) N (d) $\frac{1}{N}$

13. A random walker takes uncorrelated steps along the x-axis with probability $(P > 1/2)$ along the positive x-direction and with the probability $(1-p)$ along the negative x-direction. After N steps, the mean distance of the walker from the starting position is proportional to :

(a) $N^{1/3}$ (b) $N^{1/2}$ (c) N (d) $N^{4/3}$

14. N distinguishable particles are distributed among three states having energies 0, $k_B T$ and $2k_B T$ respectively. If the total equilibrium energy of the system is 151.23 $k_B T$, the number of particles of the system is about

(a) 152 (b) 264 (c) 356 (d) 635

ANSWER KEY

1. (d)	**2. (c)**	**3. (c)**	**4. (a)**	**5. (c)**	**6. (c)**	**7. (a)**
8. (b)	**9. (a)**	**10. (b)**	**11. (a)**	**12. (c)**	**13. (c)**	**14. (c)**